SLIDING BILAYERS
Collective stochastic resonance and shear-melting

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BASED ON

OUTLINE

• Introduction
  – Motivation: sheared soft solids
  – Earlier approaches
  – Summary of models
  – Results in brief

• Models in detail
  – A: Sheared bilayer of interacting particles
  – B: Langevin ODEs for sheared density waves

• Results and analysis in detail
  – Model A: Melting-freezing cycles
  – Model B: Generalised stochastic resonance

• Conclusion and prospect
INTRODUCTION
Motivation: sheared soft matter

\[ \text{Stress/modulus } \equiv \frac{\sigma}{G} \sim 1 \]
\[ \text{shear-rate } \times \text{ relaxation time } \equiv \dot{\gamma} \tau \sim 1 \]

Easy with colloidal matter: “onion” packings, flux lattices, colloidal crystals

Anisotropic correlations  hysteresis
shear-melting  stick-slip

Traditional measurements: rheometry, scattering
Macroscopic, three-dimensional
Average over local, incoherent phenomena

OUR FOCUS: TWO ADJACENT LAYERS
Experimental situations of interest

- Relative sliding of close-packed planes in sheared colloidal crystal: interaction in-plane > inter-plane
- Surface force shear studies with self-assembled patterned surfaces: interlayer interaction controlled by normal force
- Sheared confined dusty plasmas: Prof I Lin’s talk

2d layers, species A and B, vary AB interaction
Shear-induced melting

Ackerson and Clark 1981; Lindsay and Chaikin 1985; Stevens et al. 1991

• Sheared colloidal crystal: Low shear-rates: ordered, sharp diffraction spots
• Higher shear-rate: (distorted) liquid-like diffraction
Shear—Melting Phase Diagram

Temperature or ionic strength vs. shear rate.

Sheared crystal, sheared liquid.
EARIER APPROACHES

particle monolayer driven over passive, periodic substrate

(a) Models (b) and (c) replace the bottom surface by a Periodic Potential

(b) (c)

Tomlinson, Frenkel-Kontorova; Das, Krishnamurthy, Sood;
Frey, Nelson

Our approach: both layers dynamic, deformable
Related work – Lane formation in interdriven particles
(Dzubiella and Löwen 2002)
Summary of models and results

(A) 2d Brownian dynamics simulations, \(N + N\) particles,
\(N = 100\) to \(256\)

\[
\begin{align*}
V_{AA} &\quad A \\
F &\quad A \\
V_{AB} &\quad B \\
-V &\quad B \\
V_{BB}
\end{align*}
\]

Screened Coulomb \(V_{AA} = V_{BB}; \ V_{AB} = \epsilon V_{AA} \ \epsilon \ll 1\)

Parameters: at equilibrium \((F = 0)\), two weakly distorted triangular lattices
MAIN RESULTS OF MODEL A
For a range of values of driving force $F$

Long, irregular cycles between order and disorder
Absent if no driving and/or no noise
Detailed noneq phase diagram: later
(B) Sheared density-wave approach: Lahiri/SR 1994

Density-wave amplitude $r(t)$
Relative phase of adjacent layers (shear strain) $\theta(t)$

Effective free-energy $V(r, \theta)$

Minima of $V(r, \theta)$: $r = 0$ “liquid”; $r = r_0$ “crystalline”

Crystal favoured: $V(r = r_0, \theta = 0) < V(r = 0)$

Work of shear deformation:
$V(r = r_0, \theta \neq 0) > V(r = r_0, \theta = 0)$

Shear: drive particle in $\theta$ direction

\[ \dot{r} \sim -\frac{\partial V(r, \theta)}{\partial r} + \text{noise} \quad (1) \]
\[ \dot{\theta} \sim -\frac{\partial V(r, \theta)}{\partial \theta} + \Omega(\text{shear}) + \text{noise} \quad (2) \]
Liquid-solid free-energy difference: \textbf{without} and \textbf{with shear}
MAIN RESULTS OF MODEL B

For a range of values of driving force $\Omega$

Irregular order-disorder cycles again!
Nonequilibrium phase diagram: later
MODEL A IN DETAIL 2d Brownian dynamics simulations, \( N + N \) particles (species A and B), \( N = 100 \) to \( 256 \)

\[
\begin{align*}
\text{Length scale: mean interparticle spacing } \ell \\
\text{Time scale: diffusion time } \ell^2 / (\text{Brownian diffusivity}) \\
\text{energy scale: } k_B T
\end{align*}
\]
\[ V_{AA}(r) = V_{BB}(r) = \epsilon^{-1}V_{AB}(r) = (U_0/r) \exp(-\kappa r) \]

\[ U_0 = 1.75 \times 10^4, \quad \kappa \ell = 0.5 \]

Equilibrium \((F = 0)\), decoupled \((\epsilon = 0)\): triangular lattice

Equilibrium \((F = 0)\), coupled \((\epsilon \neq 0)\): weakly distorted triangular lattices

Dimensionless time step \(\delta t = 6.4 \times 10^{-6}\)

\[ F^* \equiv F\ell\epsilon/V_{AB}(\ell) \quad 0.6 \text{ to } 1 \]

\[ \epsilon \quad .01 \text{ to } .05 \]

Overdamped dynamics: velocity \(\propto\) total force = interparticle + driving + noise

Noise statistics: thermal equilibrium (for simplicity)
RESULTS: Model A

Keeping interaction strengths and temperature fixed, the driving force $F$ displays three threshold values $F_i, i = 1, 2, 3$.

$F < F_1$, Drift speed $v_d$ effectively pinned at zero. Initial configuration poorly ordered – anneals into macro ordered state, then stops moving.
$F > F_1$

$v_d > 0$, with a smooth onset and enhanced velocity fluctuations
$F_1 < F < F_2$ 1 and 2 components well-ordered, drifting crystals

Two-layer version of sheared crystal, stress > yield stress
$F_2 < F < F_3$ Melting Freezing Cycles

The structure factor height (averaged over 1st ring of maxima) as a function of time, $\epsilon = 0.05$
Pair correlations along drift: $g(x)$ for species 1, as time evolves.
Pair correlations transverse to drift: $g(y)$ for species 1, as time evolves.
Onset of disorder

Onset of order
Amplitude of the melting freezing cycle as a function of driving force \( F_d, \epsilon = 0.02 \).
$F > F_3$ Both components well-ordered, smoothly sliding crystals again!
$N$-particle dynamics: Nonequilibrium phase diagram
What’s going on?

Ordered → melting-freezing cycles → ordered

Competition between timescales as two arrays of particles are pushed through each other.

\( \tau_1 = a/v_d \to \) Time in which the arrays traverse 1 lattice spacing \( a \).

\( \tau_2 \to \) Timescale of relaxation(slowest) of species-1 in the potential well provided by their species-2 neighbours (and vice versa).

\( \tau_1 \gg \tau_2 \to \) the lattices of the two species have plenty of time to relax as they interpenetrate→ smooth sliding.

\( \tau_1 \ll \tau_2 \to \) each species averages over the periodic potential of the lattice of the other species→ smooth sliding.

\( \tau_1 \sim \tau_2 \to \) maximum mutual disruption of the two lattices.
Relation to freezing in periodic potentials?

When disordered:
  total disruption of order along drive direction (x)
  BUT
  some amount of order retained in transverse direction (y)
→ each species provides periodic potential along y for the other species
  → induces order along x as well.

once ordered, movement of lattices again causes disruptions
  CYCLE REPEATS!
A STOCHASTIC RESONANCE?
(B) Sheared Langevin dynamics: Lahiri/SR 1994

Density-wave amplitude $r(t)$
Relative phase of adjacent layers (shear strain) $\theta(t)$

$$\dot{r} \sim -\frac{\partial V(r, \theta)}{\partial r} + \text{noise}$$  \hspace{1cm} (3)

$$\dot{\theta} \sim -\frac{\partial V(r, \theta)}{\partial \theta} + \Omega(\text{shear}) + \text{noise}$$  \hspace{1cm} (4)

Minima of $V(r, \theta)$: $r = 0$ “liquid”; $r = r_0$ “crystalline”
Crystal favoured: $V(r = r_0, \theta = 0) < V(r = 0)$
Work of shear deformation:
$V(r = r_0, \theta \neq 0) > V(r = r_0, \theta = 0)$
Liquid-solid free-energy difference: **without** and **with shear**

![Graphs showing liquid-solid free-energy difference](http://physics.iisc.ernet.in/˜sriram)
MAIN RESULTS OF MODEL B

For a range of values of driving force $\Omega$

Irregular order-disorder cycles again!

Nonequilibrium phase diagram: NEXT
Power spectral density of cycles

\[ S(\omega) \]

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http://physics.iisc.ernet.in/~sriram
Signal/noise ratio vs noise strength
Varying the coupling: time series

\[ \alpha = 0.15 \]

\[ \alpha = 0.16 \]

\[ \alpha = 0.18 \]
Varying the coupling: probability distribution

\[ n(\rho) \]

\[ \alpha = 0.15 \]
\[ \alpha = 0.16 \]
\[ \alpha = 0.18 \]

http://physics.iisc.ernet.in/~sriram
Amplitude-phase model: Nonequilibrium phase diagram
RELATION TO LANE-FORMATION WORK
Dzubiella and Löwen 2002

Two species A and B: $V_{AA} = V_{BB} = V_{AB}$
Oppositely driven, short-range interactions

AB interfaces discouraged since (in our notation) $\epsilon = 1$
initial homogeneous mixture separates into broad lanes

Our case: $\epsilon \ll 1$ – expect lots of interface
Our observations: maximal amount of interface
perfect alternation of A and B – lanes of unit width
CONCLUSION AND PROSPECT

• Predict novel, intermittent behaviour in sheared soft solids
  – Ordered, disordered states: comparable durations
  – $3d$ structure factor won’t pick this out

• New light on mechanism underlying shear-melting
  – Shear-melted state must contain local order-disorder cycles

• Toy model for driven crystal
  – Essential physics of many-particle system
  – Generalisable beyond mean-field

• Stochastic resonance without an oscillating potential

• Experiments?

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