Time-dependent CP violation of $B_{(s)}$ decays in the PQCD approach

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Outline

Introductions & Motivation
Time-dependent CP violation
PQCD approach
Numerical Results and Discussions
Summary
We have calculated the decay channels on the left, including the branching ratios and time-dependent CP observables.

The method we use is the PQCD approach, based on $k_T$ factorization, which is successfully applied in the non-leptonic two body B meson decays.

These decay channels can provide a clear way to extract CKM angle $\gamma$. 

\[ B_s^0 \rightarrow D_s^\pm K^\mp \]

\[ B^0 \rightarrow D^\pm \pi^\mp \]

\[ B_s^0 \rightarrow D^\pm \pi^\mp \]
Ways to extract $\gamma$

The most popular way to measure $\gamma$ is through decays $B^\pm \rightarrow DK^\pm$

There are three different ways:

- **GLW**: $D \rightarrow CP$ eigenstate ($K^+K^-$, $\pi^+\pi^-$ etc.)
- **ADS**: $D^0 \rightarrow K^\pi^+$, $\bar{D}^0 \rightarrow K^\pi^+$
- **GGSZ**: $D \rightarrow$ three body ($K^0_s\pi^+\pi^-$)

Time-dependent studies of non-CP eigenstates offer a different way to extract $\gamma$. Difficulties in experiment:

- needs large sample of $B^0$ decays
- distinguish the rapid $B^0 - \bar{B}^0$ oscillations.

We calculate this kind of time-dependent CP violation, hoping to provide some guidance for experiments in the future, especially for LHCb.
Time-dependent CP violations
Three Types of CP violation

**CP violation in decay**, when the amplitude for a decay and its CP conjugate process have different magnitudes.

**CP violation in mixing**, when the two neutral mass eigenstates cannot be chosen to be CP eigenstates.

**CP violation in the interference between decays with and without mixing**, in decays into final states that are common to $B^0$ and $\bar{B}^0$. 
An arbitrary neutral b-flavored state can be described as follows:

\[ B(t) = a(t)|B^0\rangle + b(t)|\bar{B}^0\rangle \]

It satisfies the time-dependent Schrödinger equation

\[ i\frac{d}{dt} \begin{pmatrix} a \\ b \end{pmatrix} = H \begin{pmatrix} a \\ b \end{pmatrix} = (M - \frac{i}{2}\Gamma) \begin{pmatrix} a \\ b \end{pmatrix} \]

The eigenstates of the neutral b-flavored mass matrix are:

\[ |B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle, \quad |B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle \]

\[ \Delta M \equiv M_H - M_L \]
\[ \Delta \Gamma \equiv \Gamma_L - \Gamma_H \]
\[ \Gamma \equiv \frac{\Gamma_H + \Gamma_L}{2} \]
B mixing

By solving the Schrödinger equation, we can get the time evolution of an initially pure $|B^0\rangle$ or $|\bar{B}^0\rangle$ is

$$
|B^0(t)\rangle = g_+(t)|B^0\rangle + \left(\frac{q}{p}\right)g_-(t)|\bar{B}^0\rangle
$$

$$
|\bar{B}^0(t)\rangle = g_+(t)|\bar{B}^0\rangle + \left(\frac{p}{q}\right)g_-(t)|B^0\rangle
$$

where

$$
g_{\pm}(t) = \frac{1}{2} \left( e^{-iM_L t} e^{-\frac{1}{2} \Gamma_L t} \pm e^{-iM_H t} e^{-\frac{1}{2} \Gamma_H t} \right)
$$

In Standard Model, we can get

$$
\frac{q}{p} \approx \frac{V_{tq}V_{tb}^*}{V_{tq}^*V_{tb}}
$$

$$
\left(\frac{q}{p}\right)_{B_d} \approx e^{-2i\beta}, \quad \left(\frac{q}{p}\right)_{B_s} \approx e^{-2i\beta_s}
$$

$$
\beta = 21^\circ, \quad 2\beta_s = -2.5^\circ
$$
The time-dependent rate for an initially pure $B^0/\bar{B}^0$ to decay to a final state $f$:

$$\Gamma(B^0(t) \to f) = \frac{1}{2} \Gamma(B^0 \to f) e^{-\Gamma t} (1 + |\lambda_f|^2)$$
$$\times \left\{ \cosh \frac{\Delta \Gamma t}{2} - A_{\Delta \Gamma} \sinh \frac{\Delta \Gamma t}{2} + C \cos \Delta m t - S \sin \Delta m t \right\}$$

$$\Gamma(\bar{B}^0(t) \to f) = \frac{1}{2} \Gamma(B^0 \to f) \left( \frac{p}{q} \right)^2 e^{-\Gamma t} (1 + |\lambda_f|^2)$$
$$\times \left\{ \cosh \frac{\Delta \Gamma t}{2} - A_{\Delta \Gamma} \sinh \frac{\Delta \Gamma t}{2} - C \cos \Delta m t + S \sin \Delta m t \right\}$$

Then we obtain a time-dependent rate asymmetry:

$$\frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)} = \frac{C \cos \Delta m t - S \sin \Delta m t}{\cosh \frac{\Delta \Gamma t}{2} - A_{\Delta \Gamma} \sinh \frac{\Delta \Gamma t}{2}}$$
**CP Obervables**

Some definitions:

\[ C = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \]
\[ S = \frac{2 \text{Im}(\lambda_f)}{1 + |\lambda_f|^2} \]
\[ A_{\Delta \Gamma} = \frac{2 \text{Re}(\lambda_f)}{1 + |\lambda_f|^2} \]

\[ \lambda_f = \frac{q A(\bar{B}^0 \to f)}{p A(B^0 \to f)} \]
\[ \lambda_f = |\lambda_f| e^{i(\Delta - (\gamma + 2\beta) + \pi)} \]

\[ A(\bar{B}^0 \to f) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* (f | H | \bar{B}^0) \]
\[ A(B^0 \to f) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cd}^* (f | H | B^0) \]

\( \Delta \) is the strong interaction final-state phase difference. For \( B_s^0 \), there is no \( \pi \).

If the final state is the CP-conjugate state \( \bar{f} \), we have a similar formula with the following replacement: \( \lambda_f \to \lambda_{\bar{f}}, \ C \to \bar{C}, \ S \to \bar{S}, \ A_{\Delta \Gamma} \to \bar{A}_{\Delta \Gamma} \)

\[ \lambda_{\bar{f}} = \frac{q A(\bar{B}^0 \to \bar{f})}{p A(B^0 \to \bar{f})} \]
\[ \lambda_{\bar{f}} = \frac{1}{|\lambda_f|} e^{i(\Delta - (\gamma + 2\beta) + \pi)} \]
\[ \bar{C} = -C \]
How to extract $\gamma$

Because $\lambda_f$ and $\lambda_{\bar{f}}$ are observables, the weak phase can be derived from:

$$\lambda_f \cdot \lambda_{\bar{f}} = e^{-2i(\gamma+2\beta)}$$

The conventional method:

First

$$c = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}$$

Second

$$S = \frac{2|\lambda_f| \sin(\Delta - (\gamma + 2\beta))}{1 + |\lambda_f|^2} \quad \bar{S} = -\frac{2|\lambda_f| \sin(\Delta + (\gamma + 2\beta))}{1 + |\lambda_f|^2}$$
How to extract $\gamma$

We introduce the following quantities for convenience.

$$
\langle S \rangle_+ = \frac{S + \bar{S}}{2} = -\frac{2|\lambda_f| \cos \Delta \sin (\gamma + 2\beta)}{1 + |\lambda_f|^2}
$$

$$
\langle S \rangle_- = \frac{\bar{S} - S}{2} = -\frac{2|\lambda_f| \sin \Delta \cos (\gamma + 2\beta)}{1 + |\lambda_f|^2}
$$

Using the knowledge of from step one, we get

$$
s_+ = \cos \Delta \sin (\gamma + 2\beta) \quad s_- = \sin \Delta \cos (\gamma + 2\beta)
$$

Finally, we have

$$
\sin^2 (\gamma + 2\beta) = \frac{1}{2} \left[ (1 + s_+^2 - s_-^2) \pm \sqrt{(1 + s_+^2 - s_-^2)^2 - 4s_+^2} \right]
$$

The extraction of this phase suffers from an eightfold discrete ambiguity.
The Perturbative QCD Approach
We use the PQCD approach, which is based on the $k_T$ factorization. Due to the heavy mass of B, the process is dominated by the exchange of hard gluons. Thus the process is factorized into the hard part which can be calculated perturbatively and the soft part which can be absorbed into wave function which is universal and nonperturbative.
Advantage of PQCD

- This method is usually applied in the hadronic two-body decays of B meson and has achieved great success.

- The end-point singularity in collinear factorization can be avoided because of the transverse momentum.

- The nonfactorizable and annihilation diagrams are calculable in the PQCD approach.
The key step is to calculate the transition matrix elements

\[ \langle DM | H_{\text{eff}} | B \rangle \]

The weak effective Hamiltonian can be written as

For \( B \rightarrow DM \)

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub}^* V_{cd(s)} [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] \]

\[ O_1 = (\bar{b}_\alpha u_\beta)_{V-A} (\bar{c}_\beta d(s)_\alpha)_{V-A} \]

\[ O_2 = (\bar{b}_\alpha u_\alpha)_{V-A} (\bar{c}_\beta d(s)_\beta)_{V-A} \]

For \( B \rightarrow \bar{DM} \)

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud(s)} [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] \]

\[ O_1 = (\bar{b}_\alpha c_\beta)_{V-A} (\bar{\nu}_\beta d(s)_\alpha)_{V-A} \]

\[ O_2 = (\bar{b}_\alpha c_\alpha)_{V-A} (\bar{\nu}_\beta d(s)_\beta)_{V-A} \]

There are no penguin operators.
Theoretical framework

- There are several typical scales

- **W boson mass**
  - the *Wilson coefficients* $C(t)$ of the effective four-quark operators.

- **b quark mass**
  - the calculable **hard part** $H(x,b)$

- **Factorization scale**
  - nonperturbative and described by the hadronic **wave functions** of mesons, which is universal for all decay modes.
The theoretical framework

- The decay amplitude can be factorized into the convolution of the Wilson coefficients, the hard scattering kernel and the light-cone wave functions of mesons characterized by different scales.

\[ A \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \times \text{Tr} \left[ C(t) \Phi_B(x_1, b_1) \Phi_{M2}(x_2, b_2) \Phi_{M3}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right] \]
Numerical Results and Discussions
For $B^0 \rightarrow D^{\pm} \pi^{\mp}$, we take $s \rightarrow d$.
For $B_s^0 \rightarrow D^{\pm} \pi^{\mp}$, there is only annihilation diagrams

Diagrams
Here, the CKM angle $\gamma$ is an input parameter. We adopt the recent LHCb $\gamma$ average, $\gamma = 71.1^{+16.6}_{-15.7} (\degree)$.

| $B_s^0 \rightarrow D_s^+ K^{-}$ | $2.25^{+1.70}_{-0.98} \times 10^{-5}$ | $(1.9 \pm 0.12 \pm 0.13^{+0.12}_{-0.14}) \times 10^{-4}$ |
| $B_s^0 \rightarrow D_s^{-} K^{+}$ | $1.21^{+1.04}_{-0.55} \times 10^{-4}$ |
| $B^0 \rightarrow D^+ \pi^{-}$ | $1.22^{+0.74}_{-0.46} \times 10^{-6}$ | $(7.8 \pm 1.4) \times 10^{-7}$ |
| $B^0 \rightarrow D^{-} \pi^{+}$ | $2.25^{+1.61}_{-0.95} \times 10^{-3}$ | $(2.68 \pm 0.13) \times 10^{-3}$ |
| $B_s^0 \rightarrow D^+ \pi^{-}$ | $1.71^{+1.16}_{-0.70} \times 10^{-7}$ | ... |
| $B_s^0 \rightarrow D^{-} \pi^{+}$ | $1.80^{+0.86}_{-0.59} \times 10^{-6}$ | ... |

The uncertainty is from B wave function.
## Numerical Results

### CP observables

<table>
<thead>
<tr>
<th>Process</th>
<th>$C$</th>
<th>$S$</th>
<th>$A_{\Delta \Gamma}$</th>
<th>$\bar{S}$</th>
<th>$\bar{A}_{\Delta \Gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s^0 \to D_s^\pm K^\mp$</td>
<td>$-0.69^{+0.01}_{-0.01}$</td>
<td>$-0.66^{+0.22}_{-0.01}$</td>
<td>$0.30^{+0.001}_{-0.003}$</td>
<td>$-0.69^{+0.01}_{-0.01}$</td>
<td>$0.23^{+0.01}_{-0.01}$</td>
</tr>
<tr>
<td>$B^0 \to D^\pm \pi^\mp$</td>
<td>$-1.00^{+0.00}_{-0.00}$</td>
<td>$0.044^{+0.002}_{-0.001}$</td>
<td>$0.016^{+0.002}_{-0.001}$</td>
<td>$0.042^{+0.002}_{-0.002}$</td>
<td>$0.021^{+0.001}_{-0.000}$</td>
</tr>
<tr>
<td>$B_s^0 \to D_s^\pm \pi^\mp$</td>
<td>$-0.83^{+0.02}_{-0.02}$</td>
<td>$-0.052^{+0.000}_{-0.004}$</td>
<td>$0.56^{+0.03}_{-0.03}$</td>
<td>$-0.42^{+0.02}_{-0.03}$</td>
<td>$-0.37^{+0.02}_{-0.02}$</td>
</tr>
</tbody>
</table>
$B_s^0 \rightarrow D_s^{\mp}K^{\mp}$

<table>
<thead>
<tr>
<th></th>
<th>Br(theo)</th>
<th>Br(exp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s^0 \rightarrow D_s^{\mp}K^{\mp}$</td>
<td>$1.44^{+1.21}_{-0.65} \times 10^{-4}$</td>
<td>$(1.9 \pm 0.12 \pm 0.13^{+0.12}_{-0.14}) \times 10^{-4}$</td>
</tr>
</tbody>
</table>

The experimental branching ratio can thus be defined as follows

$$BR(B_s \rightarrow f)_{\text{exp}} = \frac{1}{2} \int_0^\infty \langle \Gamma(B_s(t) \rightarrow f) \rangle dt$$

The definition of the theoretical branching ratio is

$$BR(B_s \rightarrow f)_{\text{theo}} = \frac{\tau_B}{2} \left( \Gamma(B_s^0 \rightarrow f) + \Gamma(\bar{B}_s^0 \rightarrow f) \right)_{t=0}$$

The conversion between them is

$$BR(B_s \rightarrow f)_{\text{theo}} = \left[ \frac{1 - y_s^2}{1 + \mathcal{A} \Delta \Gamma y_s} \right] BR(B_s \rightarrow f)_{\text{exp}}$$

However, the difference is very small.

$$y_s = \frac{\Delta \Gamma_s}{2 \Gamma_s} = \frac{\Gamma_L^{(s)} - \Gamma_H^{(s)}}{2 \Gamma_s} = 0.088 \pm 0.014$$
\[ B^0_s \rightarrow D^\pm_s K^{\mp} \]

<table>
<thead>
<tr>
<th></th>
<th>( C )</th>
<th>( S )</th>
<th>( A_{\Delta \Gamma} )</th>
<th>( \bar{S} )</th>
<th>( \bar{A}_{\Delta \Gamma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>theory</td>
<td>−0.69</td>
<td>−0.66</td>
<td>0.30</td>
<td>−0.69</td>
<td>0.23</td>
</tr>
<tr>
<td>experiment</td>
<td>1.01 ± 0.50 ( \pm 0.23 )</td>
<td>−1.25 ± 0.56 ( \pm 0.24 )</td>
<td>1.33 ± 0.60 ( \pm 0.26 )</td>
<td>−0.08 ± 0.68 ( \pm 0.28 )</td>
<td>0.81 ± 0.56 ( \pm 0.26 )</td>
</tr>
</tbody>
</table>

1. \( |\lambda_f| = \left| \frac{q A(\bar{B}^0 \rightarrow f)}{p A(B^0 \rightarrow f)} \right| \sim \left| \frac{V^*_{cb} V_{us}}{V^*_{ub} V_{cs}} \right| \sim 1 \)

\[ |\lambda_f| = 2.32 \]

\[ C = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad \text{Easy to measure} \]

2. \( \Delta = \text{Arg} \left( \frac{\langle f | H | \bar{B}^0 \rangle}{\langle f | H | B^0 \rangle} \right) \sim 0 \)

\( \Delta = 3.0^\circ \)

If we only consider the factorizable emission diagrams (Real number).

\( \lambda_f = |\lambda_f| e^{i(\Delta-(\gamma+2\beta))} \)

\( \lambda_\bar{f} = \frac{1}{|\lambda_f|} e^{i(-\Delta-(\gamma+2\beta))} \)

\( S \sim \bar{S} \)

\( A_{\Delta \Gamma} \sim \bar{A}_{\Delta \Gamma} \)
\[ B_s^0 \to D_s^{\pm} K^{\mp} \]

\[
S = \frac{2|\lambda_f| \sin(\Delta - (\gamma + 2\beta_s))}{1 + |\lambda_f|^2}
\]

\[
\bar{S} = -\frac{2|\lambda_f| \sin(\Delta + (\gamma + 2\beta_s))}{1 + |\lambda_f|^2}
\]

\[
A_{\Delta\Gamma} = \frac{2|\lambda_f| \cos(\Delta - (\gamma + 2\beta_s))}{1 + |\lambda_f|^2}
\]

\[
\bar{A}_{\Delta\Gamma} = \frac{2|\lambda_f| \cos(\Delta + (\gamma + 2\beta_s))}{1 + |\lambda_f|^2}
\]
\[
\mathbf{B}^0 \rightarrow \mathbf{D}^{\pm} \pi^{\mp}
\]

\[s \rightarrow d\]

<table>
<thead>
<tr>
<th>( \text{Br(theo)} )</th>
<th>( \text{Br(exp)} )</th>
<th>( C )</th>
<th>( S )</th>
<th>( A_{\Delta\Gamma} )</th>
<th>( \bar{S} )</th>
<th>( \bar{A}_{\Delta\Gamma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^0 \rightarrow D^+ \pi^- )</td>
<td>( 1.22 \times 10^{-6} )</td>
<td>( (7.8 \pm 1.4) \times 10^{-7} )</td>
<td>( -1.00 )</td>
<td>0.044</td>
<td>0.016</td>
<td>0.042</td>
</tr>
<tr>
<td>( B^0 \rightarrow D^- \pi^+ )</td>
<td>( 2.25 \times 10^{-3} )</td>
<td>( (2.68 \pm 0.13) \times 10^{-3} )</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

\[
| \lambda_f | = 42.84 \\
\Delta = 3.4^\circ
\]

\[
| \lambda_f | \sim \frac{|V_{cb}^* V_{ud}|}{|V_{ub}^* V_{cd}|} \sim \frac{1}{\lambda^2}
\]

\[
C \sim -1
\]

hard to measure

\[
S \sim \bar{S} \\
A_{\Delta\Gamma} \sim \bar{A}_{\Delta\Gamma}
\]

<table>
<thead>
<tr>
<th>( a )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>theory</td>
<td>( -0.043 )</td>
</tr>
<tr>
<td>experiment</td>
<td>( -0.030 \pm 0.017 )</td>
</tr>
</tbody>
</table>

\[
a = -\frac{s + \bar{s}}{2}
\]

\[
c = -\frac{s - \bar{s}}{2}
\]
\[ B_{S}^{0} \rightarrow D^{\pm} \pi^{\mp} \]

| \( B_{S}^{0} \rightarrow D^{+} \pi^{-} \) | 1.71 \times 10^{-7} | -0.83 | -0.052 | 0.56 | -0.42 | -0.37 |
| \( B_{S}^{0} \rightarrow D^{-} \pi^{+} \) | 1.80 \times 10^{-6} | |

Only annihilation diagrams

Branching ratios are very small

Big strong phase difference \( \Delta = 63^\circ \)

\[ |\lambda_f| \sim \left| \frac{V_{cb}^* V_{us}}{V_{ub}^* V_{cs}} \right| \sim 1 \]

\| \lambda_f \| = 3.24

Easy to measure \( C \)
$B^0 \rightarrow D^\pm a_2^\mp (1320)$

- There are 6 types of diagrams contributing to $B \rightarrow DT$ decays. Because the tensor meson can not be produced through local $(V \pm A)$ current and $(S \pm P)$ density.

![Diagram](image)

- Nonfactorizable emission diagrams
- Factorizable annihilation diagrams
- Nonfactorizable annihilation diagrams
Although $A(B^0 \to D^+ a_2^-)$ contain a small CKM matrix, $A(\bar{B}^0 \to D^+ a_2^-)$ is also depressed because there is no factorizable emission diagram with $a_2^+$ emitted ($a_2^+$ is a tensor).

Maybe $|\lambda_f|$ will not be so big as the decay $B^0 \to D^{\pm} \pi^\mp$. 

$$|\lambda_f| = \left| \frac{q A(\bar{B}^0 \to D^+ a_2^-)}{p A(B^0 \to D^+ a_2^-)} \right|$$
\[ B^0 \to D^\pm a_2^\mp (1320) \]

<table>
<thead>
<tr>
<th>[ B^0 \to D^+ a_2^- ]</th>
<th>[ B^0 \to D^- a_2^+ ]</th>
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<tbody>
<tr>
<td>[ \text{Br(theo)} ]</td>
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</tr>
<tr>
<td>[ 1.54^{+0.95}_{-0.59} \times 10^{-6} ]</td>
<td>[ 5.63^{+3.06}_{-2.02} \times 10^{-4} ]</td>
</tr>
<tr>
<td>[ C ]</td>
<td>[ S ]</td>
</tr>
<tr>
<td>[ -0.99 ]</td>
<td>[ 0.098 ]</td>
</tr>
</tbody>
</table>

\[ |\lambda_f| = 19.09 \]
\[ \Delta = 44^\circ \]

\[ |\lambda_f| \] is smaller than that of \[ B^0 \to D^\pm \pi^\mp \], but not small enough.

The reason is that non-factorizable emission diagrams and annihilation diagrams make a large contribution, which is consistent with the large \[ \Delta \].
We have calculated the branching ratios and time-dependent CP observables of $B_s^0 \rightarrow D^\pm K^\mp$, $B^0 \rightarrow D^\pm \pi^\mp$, $B_s^0 \rightarrow D^\pm \pi^\mp$ decays in the PQCD approach, providing theoretical expectation which will be useful for experiment measurements.

We find $B_s^0 \rightarrow D_s^\pm K^\mp$ is the most wonderful channel to extract $\gamma$, because it has a large branching ratio (easy to measure), and a proper $|\lambda_f|$ (experiment resolution).

Other similar decay channels can be calculated in the following work. PP mode to PV, PT mode etc.

\[ D_{(s)} \rightarrow D^*_{(s)} \]
\[ K \rightarrow K^*, \pi \rightarrow \rho \]