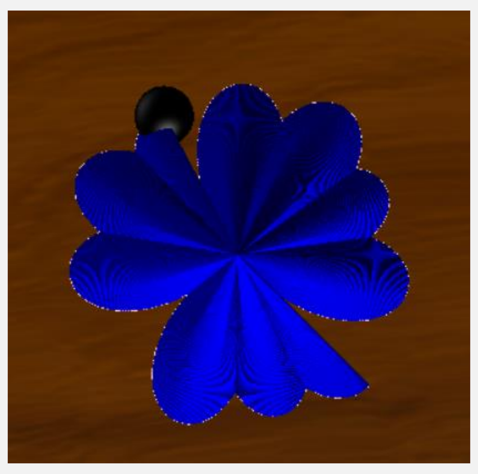
A new method on 3D damped Foucault pendulum



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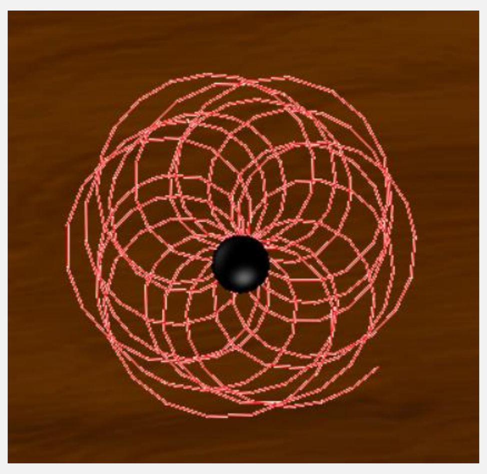

Introduction

It was long-believed that Earth rotates, but it was not until the mid-19th were it clearly demonstrated by the French physicist Léon Foucault with his Foucault pendulum. Precession of the pendulum directly proved the Earth's rotation, and it was also discovered that the precession rate depends on the local latitude.

In our study, we conceive an approach to describe the pendulum's motion in a simpler way and find it agrees with our experimental result. Furthermore, we also find out our local latitude through the measurement of the precession angle of the pendulum.

Experimental Setup

There are two bearings embedded in our pendulum support. One enables the pendulum's oscillation, and the other let it rotate. Lubricating oil is used to reduce the mechanical friction in the device, and the pendulum is initially tied to a fixed table by a cotton thread and begins its motion by burning the thread so as to minimize the fluctuations. Two cameras are used for recording the pendulum's x-y and y-z motion respectively.

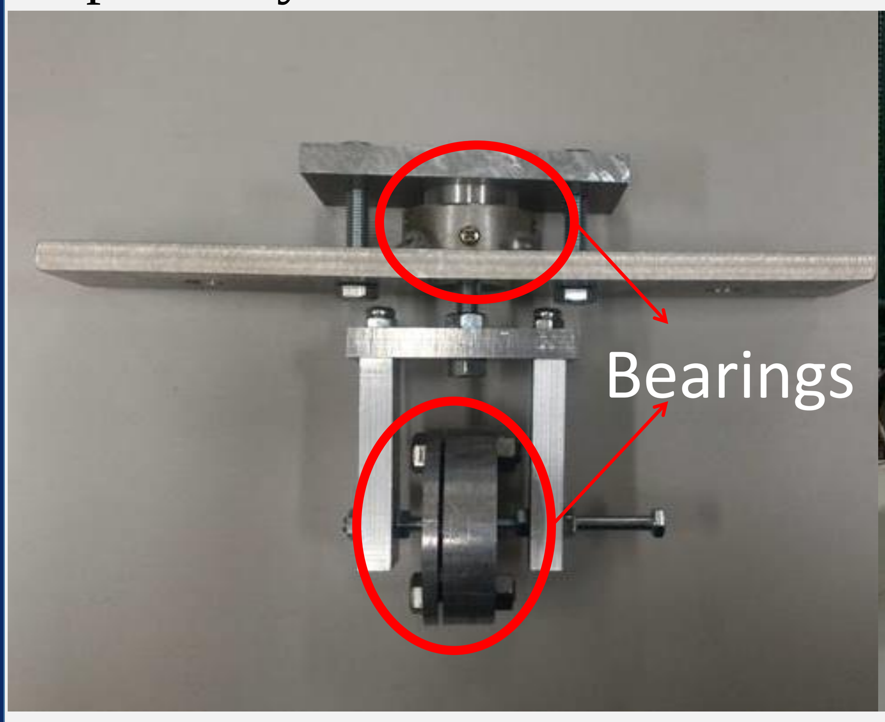

Figure1. Pendulum support consists of two bearings that makes the oscillation and rotation of the pendulum possible.

Figure2. The pendulum is tied to the table by the cotton thread (white arrow) to setup its initial conditions.

Lagrangian and coordinate transform

Euler equation (EE) with dissipating force is given as:

$$\frac{\partial}{\partial t}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^i}\right)-\frac{\partial \mathcal{L}}{\partial x^i}+\frac{\partial P}{\partial \dot{x}^i}=0$$

- $\mathcal{L}=\frac{1}{2}m_B\ell^2\dot{\theta}^2+\frac{1}{2}I_L\dot{\theta}^2-m_Bg\ell(1-\cos\theta)$: Lagrangian
- $P=\frac{1}{2}k_L\ell^2\dot{\theta}^2+\frac{1}{2}k_B\ell^2\dot{\theta}^2$: Dissipation function
- x^i: spatial coordinates with $i=1,2,3$.
- m_B: pendulum's mass. ℓ: cable's length.
- θ:oscillation angle. g: gravitational constant.
- k_L & k_B: damping constants of bearing and pendulum.

Using Laplace transform and a coordinate transform gives:

$$\begin{cases} x(t)=\ell\sin\theta\cos\varphi \\ y(t)=\ell\sin\theta\sin\varphi \\ z(t)=h-\ell\cos\theta \end{cases}$$

where

$$\theta(t)=e^{-\frac{k_B+k_L}{2\left(m_B+\frac{I_L}{\ell^2}\right)}t}\left[\theta_0\cos(\omega t)+\left(\dot{\theta}_0+\frac{k_B+k_L}{2\left(m_B+\frac{I_L}{\ell^2}\right)}\theta_0\right)\frac{\sin(\omega t)}{\omega}\right]$$

$$\omega^2=\frac{g}{l}\left(\frac{1}{1+\frac{I_L}{\ell^2}}\right)-\left(\frac{k_B+k_L}{2\left(m_B+\frac{I_L}{\ell^2}\right)}\right)^2$$

$$\varphi=\arctan\left(\frac{y}{x}\right)=\arctan\left(\frac{y_{F0}}{x_{F0}}\right)=constant$$

h is the height of the pivot.

The above equations only describe simple pendulum. Next, by using a rotational matrix, we obtain the behavior of the Foucault pendulum:

$$\begin{bmatrix} x_F \\ y_F \\ z_F \end{bmatrix}=\begin{bmatrix} \cos\Omega t & \sin\Omega t & 0 \\ -\sin\Omega t & \cos\Omega t & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where $\Omega=\omega_E\, sin\,\lambda\cong 3.03\times 10^{-5}$ (rad/s) is the rotational angular frequency of the pendulum.

Result and discussion

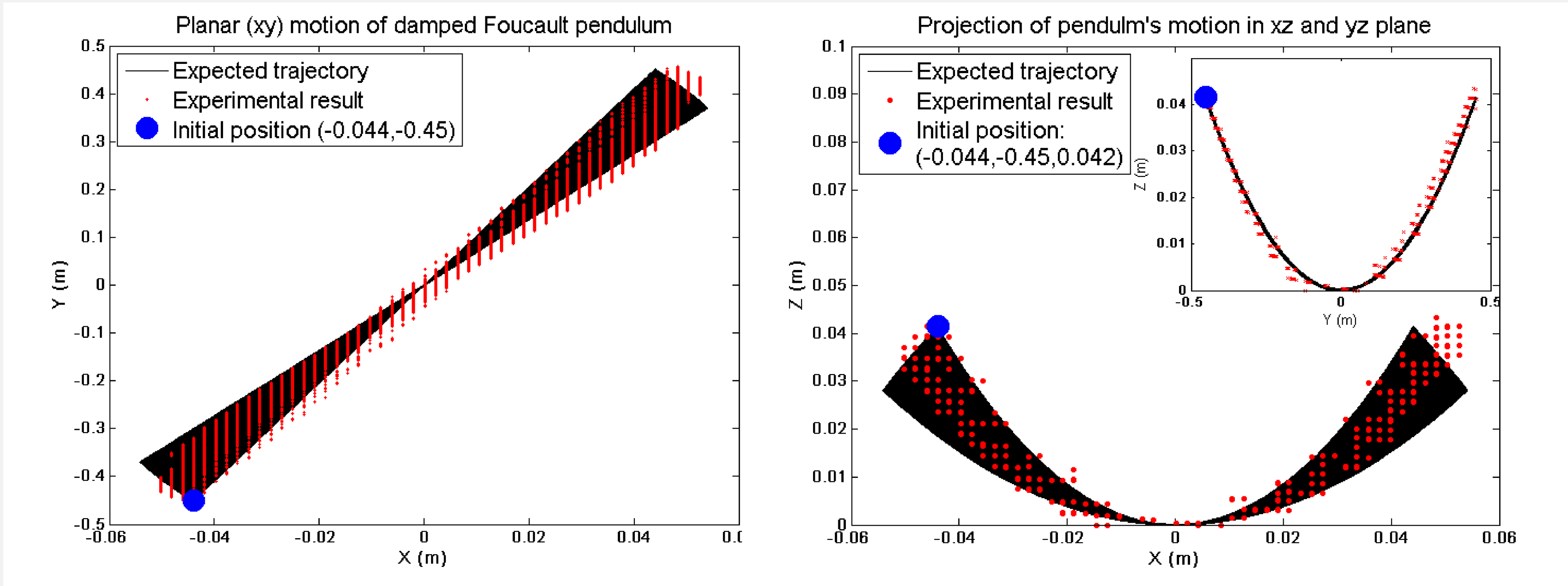

Figure 3a (left) and figure 3b (right). Planar projection of Foucault pendulum.

From Fig. 3a, with the precession angle ($\phi=2.62°$) and the time duration (τ =26.6 min), we obtain our local latitude (λ) as:

$$\lambda=\sin^{-1}\left(\frac{\phi}{2\pi}\frac{T_E}{\tau}\right)=23.2°\text{N}$$

which deviates from the reference latitude 24.6°N by 5.4%. Note that $T_E=86400$ sec., or 1440 min.

The geometries shown in Fig.3b are simply the consequence of local measurement of the pendulum's motion on xz- and yz-plane. However, its general geometry depends on initial condition and time duration.

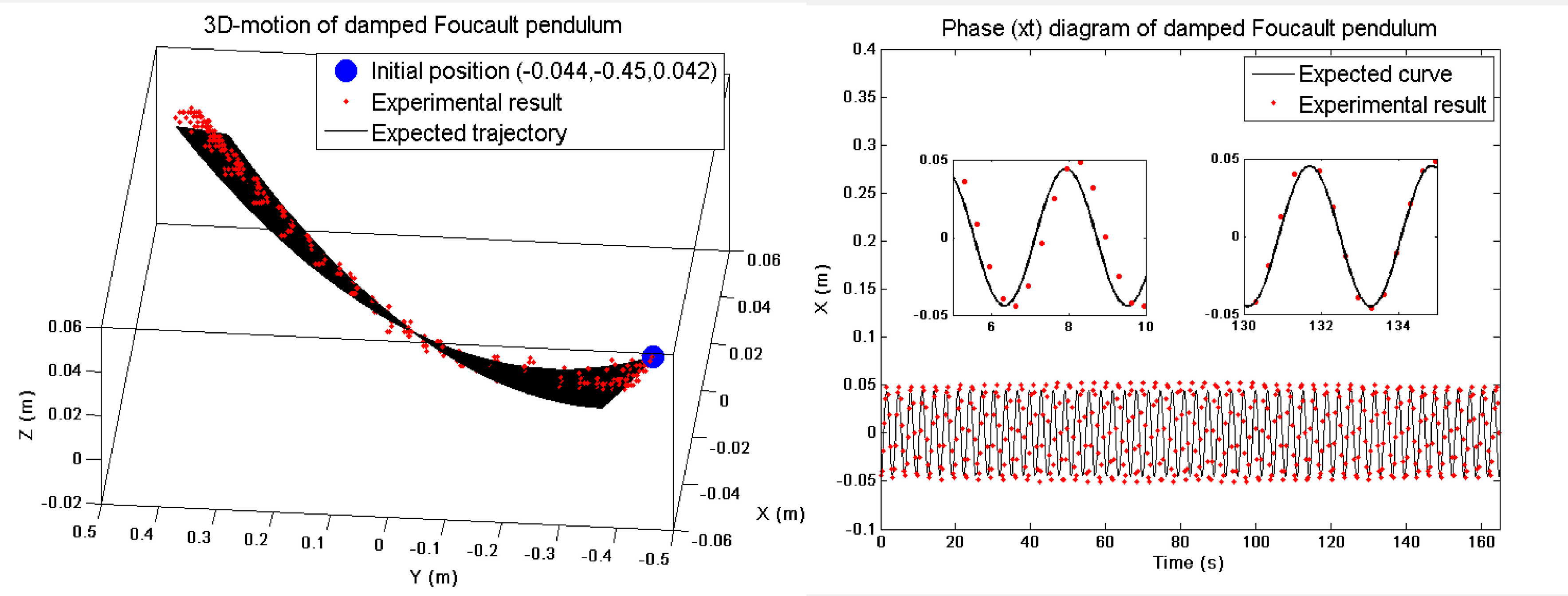

Figure 4 (left) and figure 5 (right). 3D-reconstruction and simulation of Foucault pendulum and their phase diagram.

It is shown, Fig.5, that the actual phase of x, in fact, all phases, periodically coincides and leaves the predicted curve. This is the consequence of the property of the elastic cable used in our experiment to hang the pendulum.

Conclusion

- Most discussions on Foucault pendulum focus on its xy-motion by ignoring the z-motion so as to simplify the analysis. However, with our method, we can describe the pendulum's 3D-motion much simpler and without regarding to fictitious forces.
- The oscillation period is constant even when damping is taken into account and it is measured to be 3.27 sec., whereas it is expected to be 3.17 sec. from theoretical analysis.
- The latitude we measured by using the precession angle and time duration of the experiment gives us the result of 23.2°N.

Reference

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