Spontaneous Knotting of An Agitated String

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Introduction

It is well known that agitated string tends to get knotted spontaneously. When the string is shaken the arrangement of the string will become more complicated, and forming many holes. These holes may have probability to form the knots. In our study, we examine the factors that affect the spontaneous knotting probability by tumbling the string under different conditions.

Experimental Setup

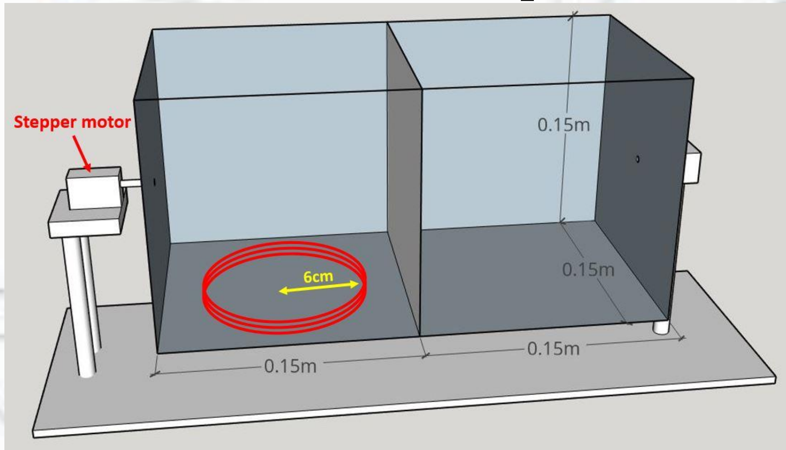

Fig. 1 schematic diagram of experimental setup.

We design two cubic boxes each with 0.15m side length and use a stepper motor to rotate the boxes. In addition, the string is initially setup to be enclosed within a radius of 6 cm.

Results and Discussions

There are two parameters we changed in our experiment. That is, the rotation rounds and the string length. Knotting probability of our experiment results are shown in Fig 2.

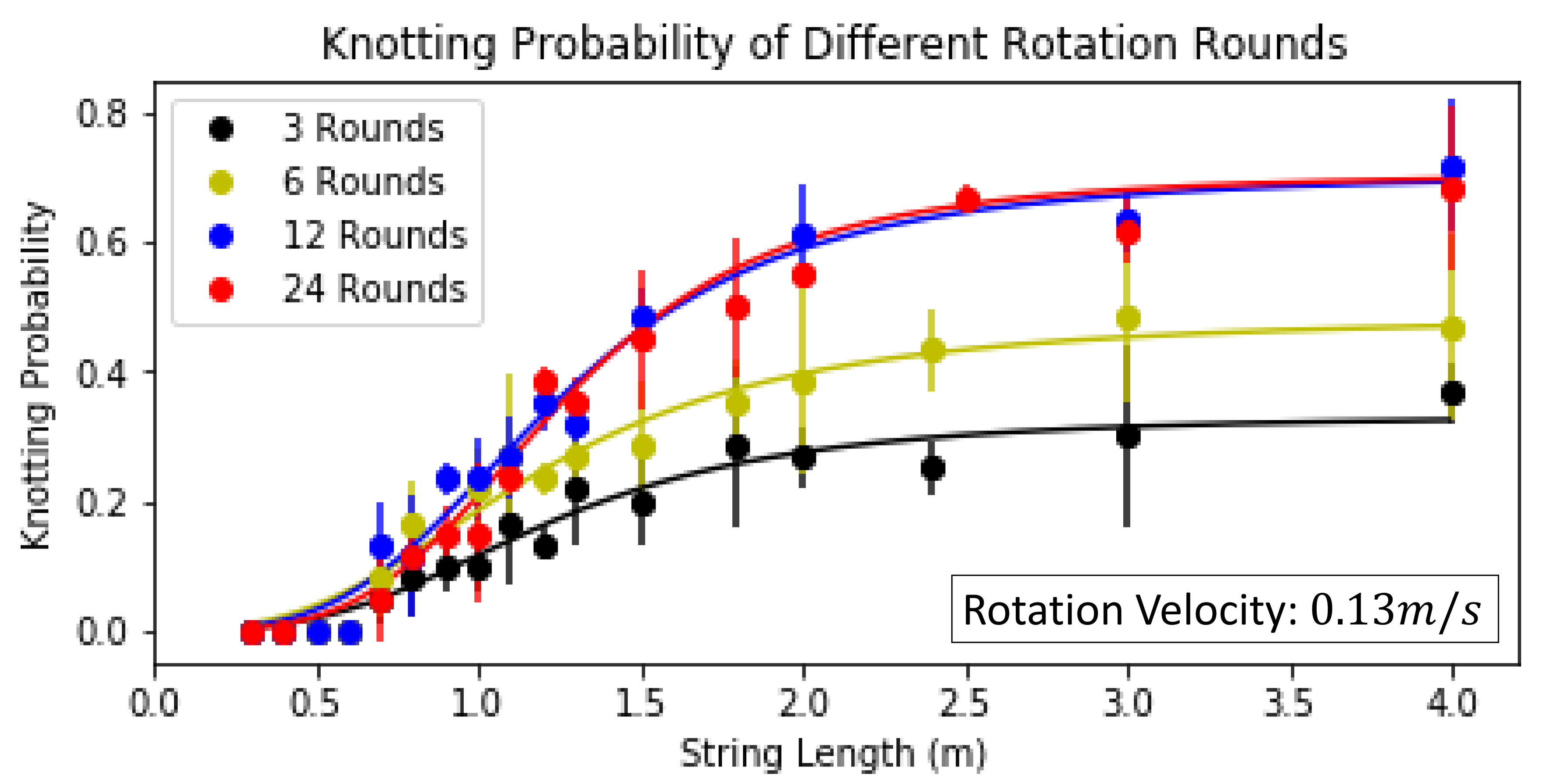

Fig. 2 Knotting probability of different rotation rounds for different lengths.

The effective volume is associated with the velocity of rotation, as the velocity grows, L_0 grows (Fig. 4).

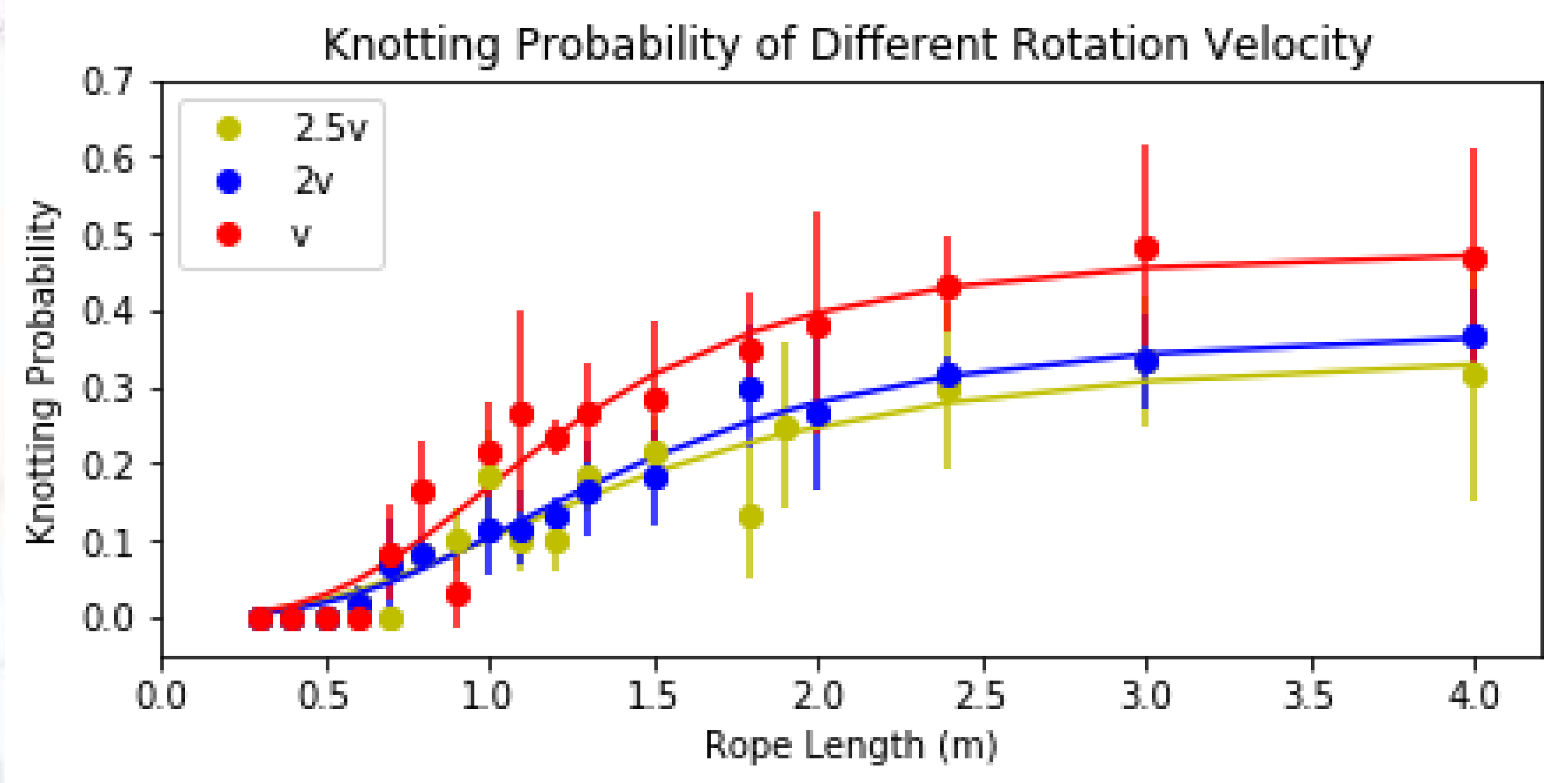

Fig. 4 L_0 for rotation velocity v, $2v$, and $2.5v$ is respectively $1.2m$, $1.4m$, and $1.4m$, where $v = 0.13m/s$.

Changing String Length

For a simple explanation, we reduce 3D knotting probability into 2D. Knotting may occur when the ends of the string fall into the effective region. The maximum effective region is produce by the string of length longer than L_0. The equivalent knotting area is the superposition of the knotting area from each layer. As the string length increases, the knotting area increases (Fig. 3).

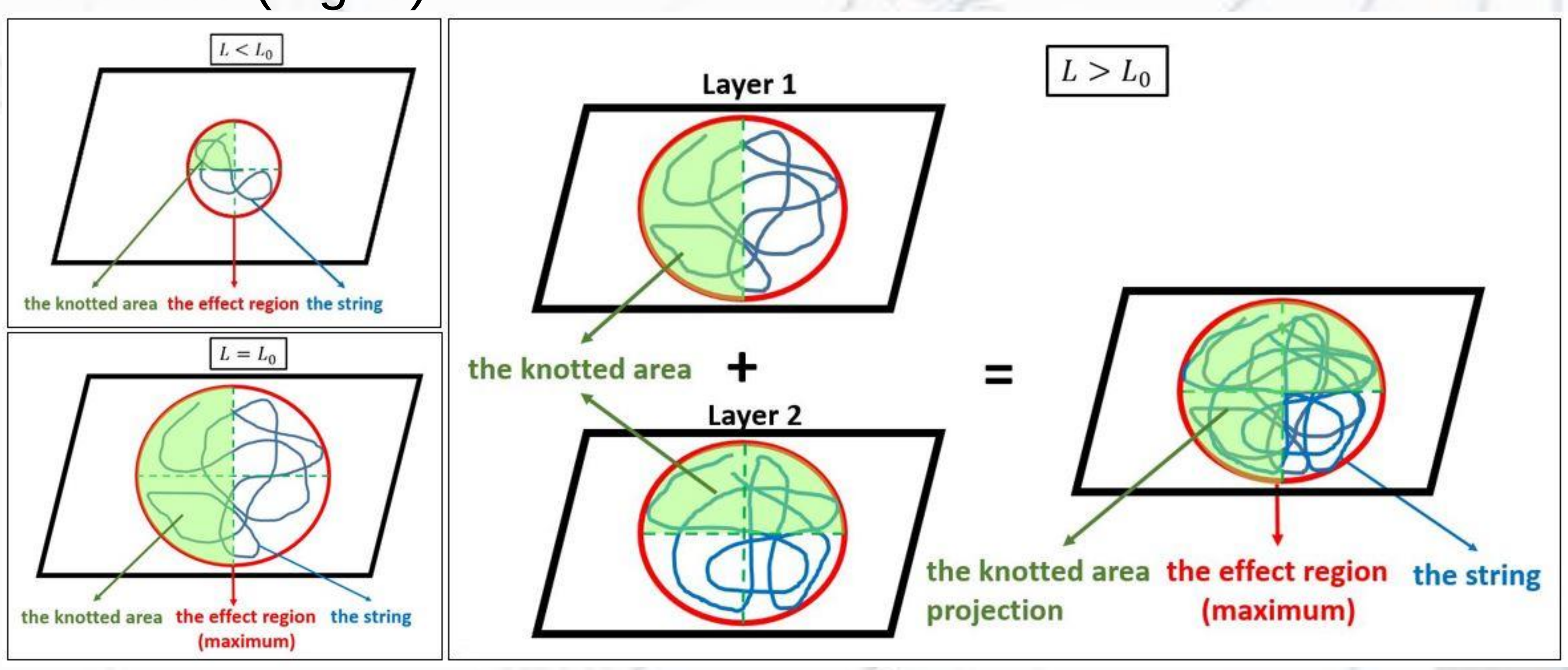

Fig. 3 Schematic diagram of 2D knotting probability for different length scales of the string. In fact, our experiment is 3D.

We suppose the knotting probability is the product of a saturated probability and the ratio of knotted volume(L^3) and effective volume ($L^3 + {L_0}^3$). That is,

$$P = P_0\left(\frac{L^3}{L^3+{L_0}^3}\right) = P_0\left(1+\left({}^{L_0}/_{L}\right)^3\right)^{-1}. \quad (1)$$

Note that $({}^{L}/_{L_0})^3$ indicates the complexity of the winding. The larger it is, the higher the chance of knotting.

Changing Rotation Time

As Fig. 5 shows. For a given string length, the overall knotting probability does increase as the round increases. Nevertheless, it asymptotes a value less than unity. This is due to the complexification of the string's geometric structure through the knots. As the structure gets complexities, it gets more difficult to alter the configuration. Therefore, the overall probability converges to a value less than unity. By fitting, the overall probability is given by

$$P_0 = P_c(1-e^{-r/R}), \quad (2)$$

where P_c is convergent probability, r is rotation rounds, and R is time constant.

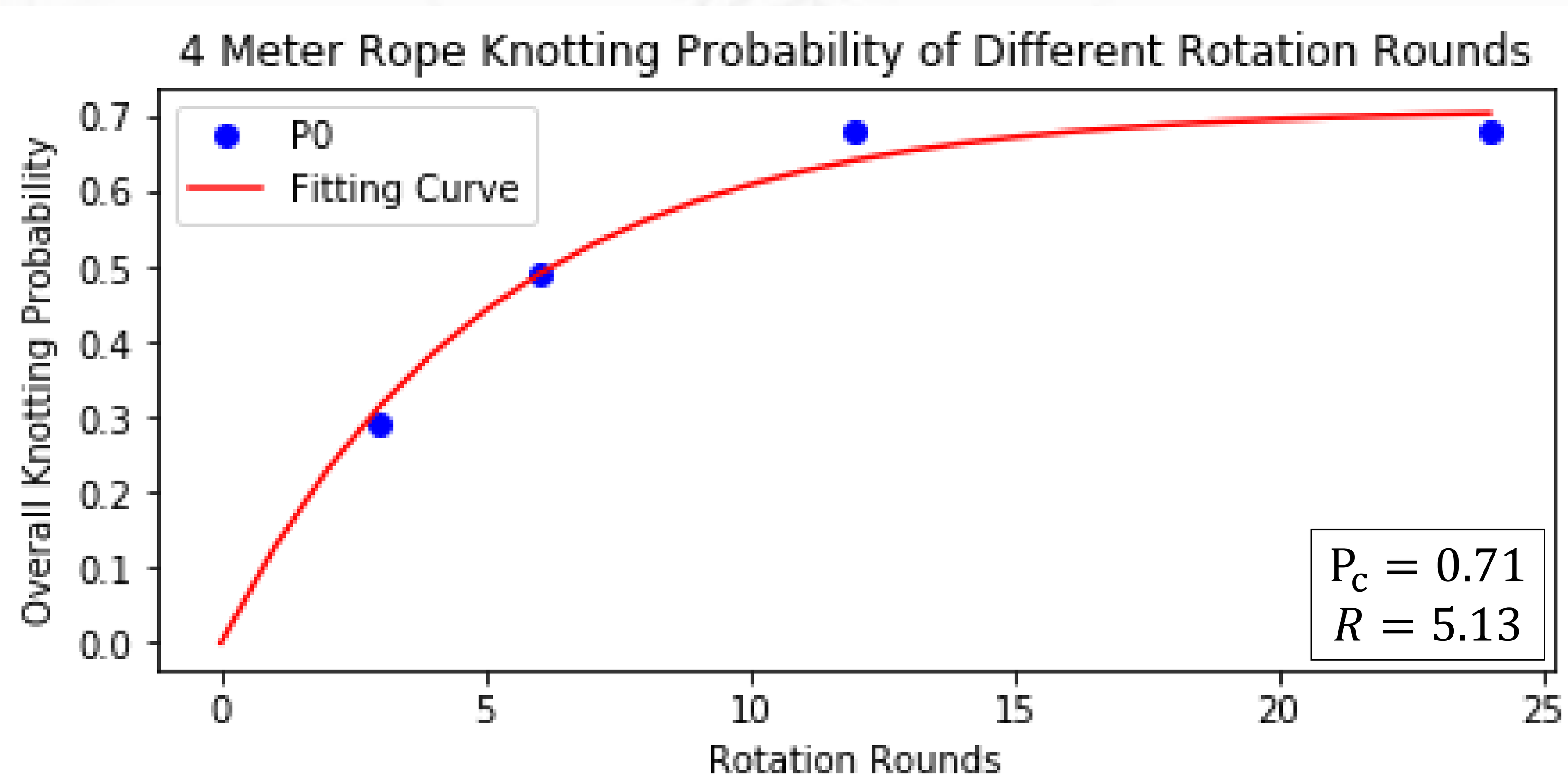

Fig. 5 Knotting probability events occurring after a rotation of r rounds.

Conclusions

1. The probability of knotting grows with length but converges to the ratio of the effective volume and the volume of box, and when $L = L_0$, the probability of knotting is $1/2$.
2. The probability of knotting grows with time but converges to a value $P_c < 1$ because the structure of string can be fixed without a real knot.

Reference

[1] Dorian M. Raymer and Douglas E. Smith, National Academy of Sciences, 17911269 (2007)