Relationship of Faraday Waves Pattern and Parameters

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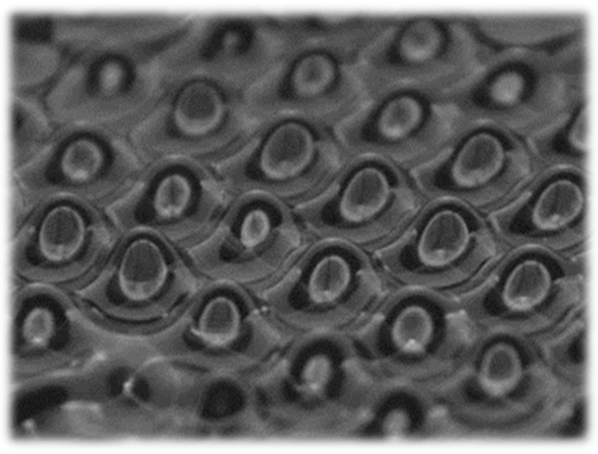
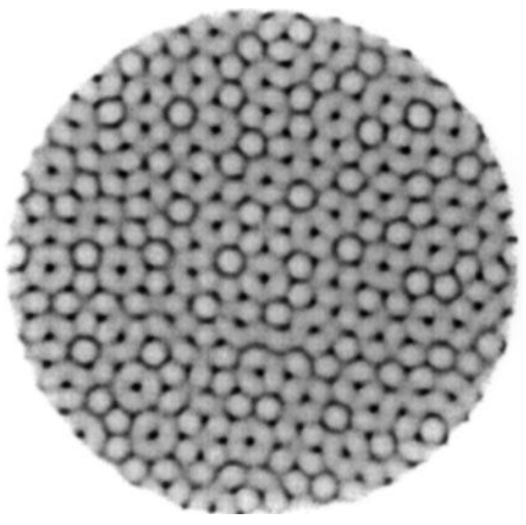
Abstract

When giving a critical amplitude, nonlinear standing waves appear on liquids. It is called Faraday waves. The viscosity and depth of fluid, the shape and size of the container or the amplitude and frequency of the supplied wave could lead to different graph. We prepare round container. We focus on the pattern and explore the wave numbers difference when changing frequency. Shake round container would produce a ring pattern, we focus on the relationship between the wavelength and the frequency.



▲ Figure 1 faraday wave (side view)



▲ Figure 2 faraday wave (top view)

Introduction

Faraday wave is a kind of standing wave which appears on the surface of fluid, which is the result of the parameter resonance in the fluid. We need to introduce the equation of fluid motion

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u} + g_z(t)\hat{e_z} \qquad ①$$

with $\boldsymbol{u}$ the velocity field, p the pressure, ρ and ν the density and kinematic viscosity of the fluid, respectively, and $g_z(t) = -g - f\cos\omega t$ the effective gravity.

We assume that it is ideal fluid, so the viscosity coefficient could be neglected. We use Eulerian equation of motion and the continuous equation of the fluid and substitute the boundary condition:

$$\frac{\partial\phi}{\partial n} = 0 \text{ (at container boundary)} \qquad ②$$

$$\frac{\partial\phi}{\partial z} = 0 \text{ (z = h) , h : liquid depth} \qquad ③$$

to formulate Mathieu's equation:

$$\frac{d^2 a_m}{dT^2} + (p_m - 2q_m\cos 2T)a_m = 0 \qquad ④$$

with:

a_m: eigenvalue of eigenfunction

$$p_m = \frac{4k_m \tanh k_m h}{\omega^2}\left(g + \frac{{k_m}^2\gamma}{\rho}\right) \quad \text{(influenced by frequency)}$$

$$q_m = \frac{2k_m f \tanh k_m h}{\omega^2} \quad \text{(influenced by amplitude)}$$

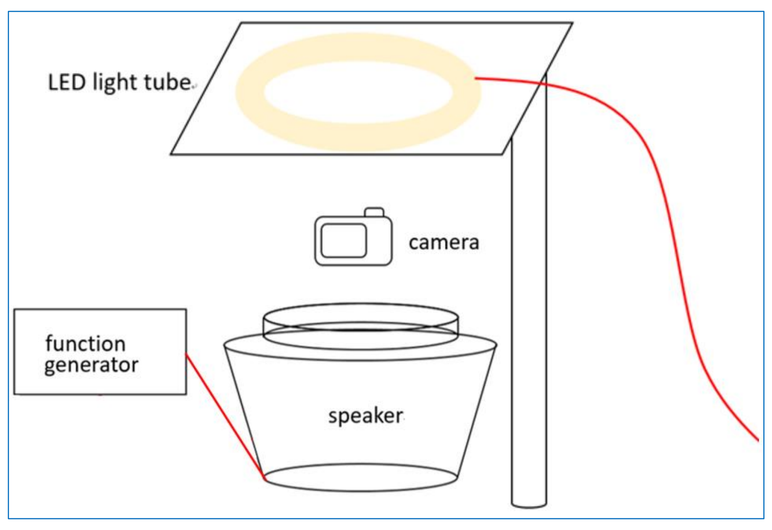
If we neglect the amplitude intensity, then $f = 0$, $q_m = 0$, Mathieu's equation can be thought simple harmonic motion ,thus the relationship between ω and k :

$$\frac{\omega_m}{2\pi} = \frac{1}{2\pi}[(\tanh k_m h)(\frac{{k_m}^3\gamma}{\rho} + k_m g)]^{\frac{1}{2}} \qquad ⑤$$

Method

Set up

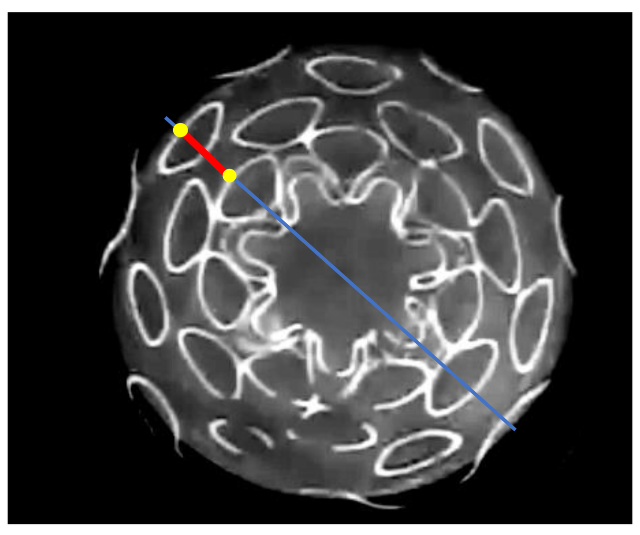
(1) Use a speaker, a round container and a function generator to form a vibration flat.
(2) Set up a light source by LED light tube.
(3) Record the waveform by a high speed camera.



▲ Figure3 Procedure of experiment

Wave length in a round container

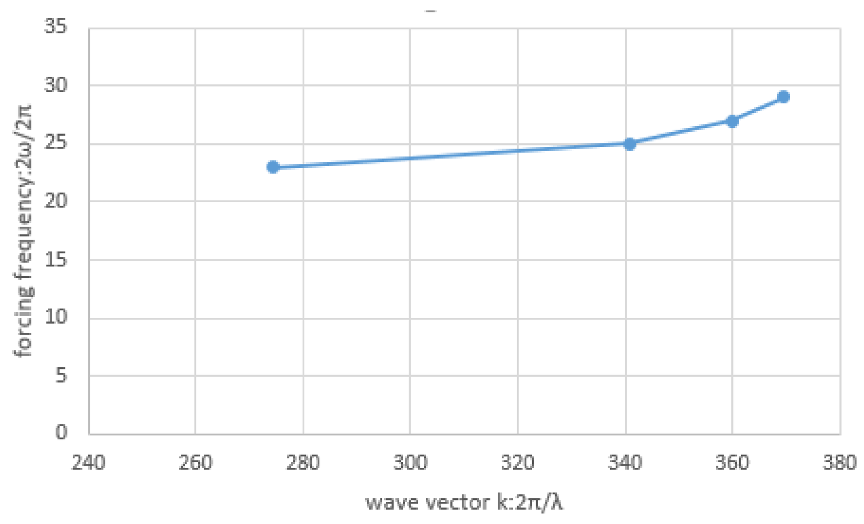
To get a accurate wave length, we need to transform the picture to greyscale then and use image processing to measure the wavelength.
In figure 4, the wave length (red line) is the length between two lattice (yellow points). Multiple by the ratio of 12cm (real diameter) to the pixel value of the blue line. To avoid the effect of light, we take the average of all direction wavelength.



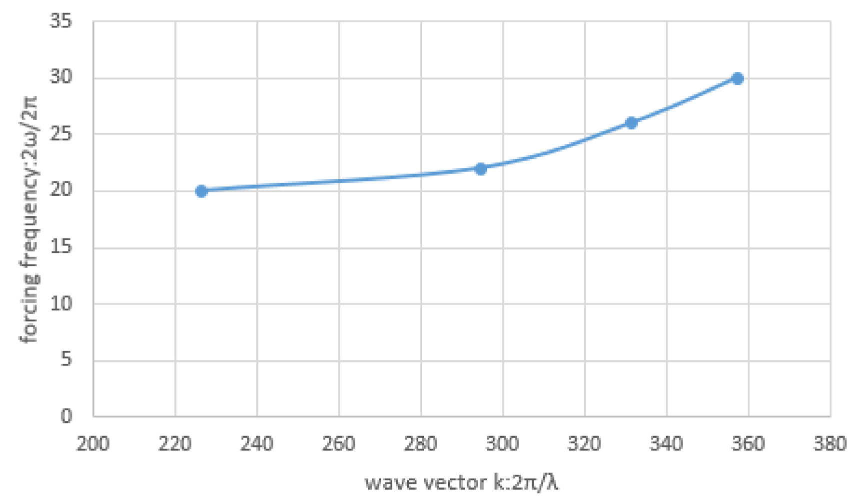
▲ Figure4 The graph in round container(grey scale)

Result

When we fixed the depth of water in 0.7cm in round container and adjusted the supply frequency. The wavelengths we got are shown in figure 5. Then we get another data with the depth 1cm.The wavelengths we got are shown in figure 6.

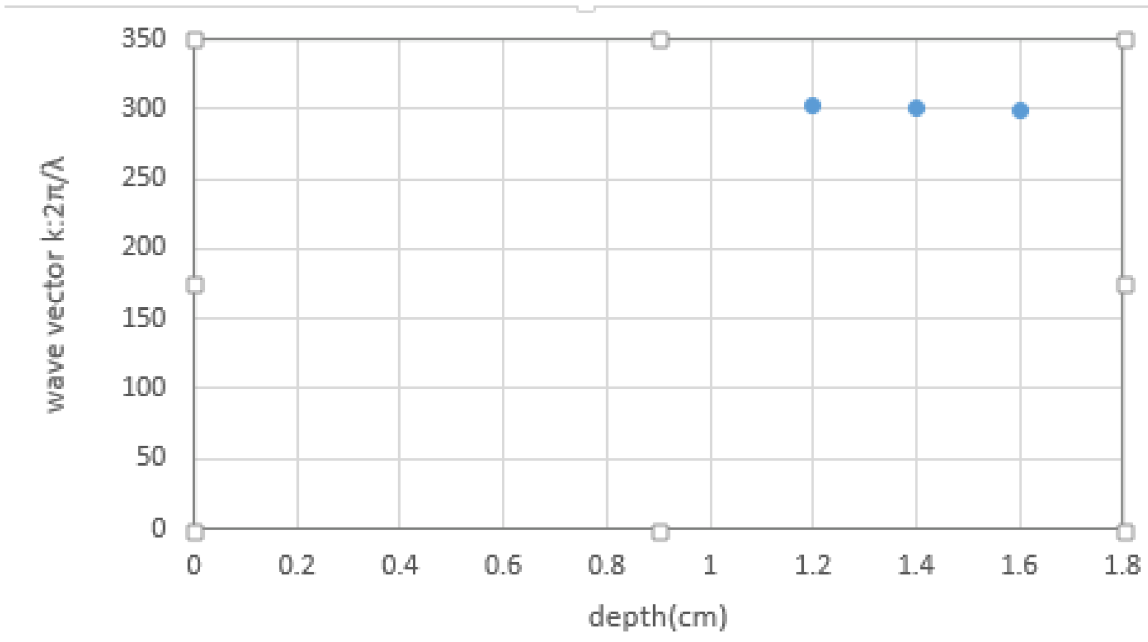


▲ Figure 5 The result with the depth 0.7cm



▲ Figure 6 The result with the depth 1cm

Then we fixed the frequency and changed the depth of the water, we get another data:



▲ Figure 7 The result of changing depth

Conclusion

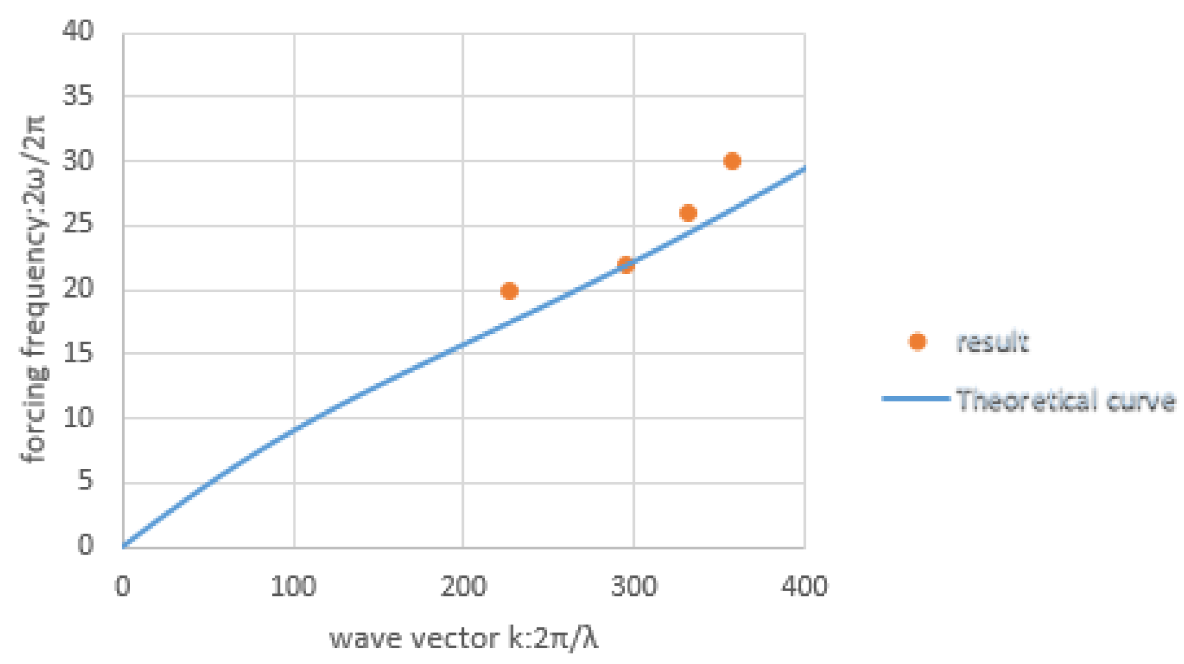
Traditional analysis of wavelengths can only work when giving high frequency and lattice wave pattern was generated. We use image processing to measure the distance from one brightness peak to the another.

We find the relationship function among frequency, depth, wavelength, etc. In the first step, we could know that amplitude has no significant effect on the faraday pattern in our experiment. Then we focus on the frequency. So we verify this function

$$\frac{\omega_m}{2\pi} = \frac{1}{2\pi}[(\tanh k_m h)(\frac{{k_m}^3\gamma}{\rho} + k_m g)]^{\frac{1}{2}}$$

by our experiment result.

The wavelength vs. frequency graph under the condition:
h = 1 cm , γ = 0.07275 dyne/m , g = 9.8 m/s², ρ = 1000 kg/m³ ,
the orange points are our experiment data of changing frequency, and the blue line is theoretical curve.



▲ Figure 8 The result and theoretical curve

Through the function ⑤ and our experiment data, we could know that frequency has more influence on the wave number than the depth since the parameter h is wrapped in hyperbolic tangent, and when we rise frequency will increase wave vector ,this property follows the function ⑤ .

However ,there are some errors in our data, if we increase the amount of data may be able to avoid this error.

Reference

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