Proton Stability in Low-Scale Extra-Dimensional Grand Unified Theories

Mitsuru Kakizaki (University of Toyama)

1. Motivation

2. Localizing Fermions in Extra Dimensions (Review)

3. Localizing X-bosons and Proton Stability

4. Summary
1. Motivation

- Discovery of a Higgs boson at the CERN LHC
  - The Standard Model (SM) is established as a low-energy effective theory below $O(100)$ GeV

- Theoretical problems of the SM include
  - Unexplained structure of the SM gauge bosons and fermions
  - Quantization of the electric charges
  - Fermion mass hierarchy

We need a more fundamental theory

- Grand unified theories (GUTs)
  - Unification of gauge groups: $SU(3)_C \times SU(2)_L \times U(1)_Y \subset G$
  - Unification of quarks and leptons: $(Q,U^C,E^C) = 10 \quad (D^C,L) = 5^*$

Prediction: Proton is not absolutely stable
This talk

- **Proton Decay Constraint**
  \[ \tau_p(p \rightarrow e^+\pi^0) > 8.2 \times 10^{33} \text{ yr} \]
  [Super-Kamiokande, 2009]
  \[ M_X / g_X \geq 10^{15} \text{ GeV} \]

- **Ordinary GUTs:**
  \[ g_X \sim 1 \quad M_X \sim 10^{14-16} \text{ GeV} \]
  Far beyond reach of collider experiments

- **This work**
  - New mechanism to localize X-bosons in extra dimensions
  - Localizing the X-bosons and fermions at different locations
  Proton decay is extremely suppressed

The GUT scale can be \[ M_X \sim O(10) \text{ TeV} \]
2. Localizing fermions in extra dimensions

- **Idea:**
  Chiral fermions are localized due to a kink in extra dimensions

  [Jackiw, Rebbi, 1976; Rubakov, Shaposhnikov, 1983; Arkani-Hamed, Schmaltz, 2000]

- **$Z_2$-invariant real scalar theory in 5D space-time $x^M = (x^\mu, y)$:**

  $$ S = \int d^4x \, dy \left[ \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} (\partial_y \phi)^2 - V(\phi) \right] $$

  $V(\phi)$: Double well

  Classical domain wall solution:

  $$ \langle \phi \rangle (y \to \pm \infty) = \pm v $$

  Around the origin: $\langle \phi \rangle \propto y$

- **Translational invariance along the extra dimension is broken**
Profile of a fermion wave function

- 5D Fermion $\Psi$ in the domain wall background $\langle \phi \rangle = 2\mu^2 y$:

$$S = \int d^4x dy \bar{\Psi} \left[ i\gamma^\mu \partial_\mu - \gamma^5 \partial_y + (\phi + m) \right] \Psi$$

Left chiral zero mode in 4D:

$$\Psi^0 \sim \sqrt{\mu} \exp[-\mu^2(y - l)^2] \psi_L(x), \quad l \equiv -\frac{m}{2\mu^2}$$

- The zero mode wave function $\Psi^0$ is localized at the zero of $\phi + m$ with a Gaussian profile
- The right-handed zero mode is killed
3. Localizing X-bosons and Proton Stability

- Idea:
  Kink background of an adjoint Higgs
  Different profiles in the same GUT multiplets

- $Z_2$-invariant real $\Sigma(24)$ terms in 5D SU(5) GUT Lagrangian:

$$\mathcal{L}_5 = \text{tr}(\partial^M \Sigma)(\partial_M \Sigma) + \frac{M_5^2}{2} \text{tr} \Sigma^2 - \frac{a}{4} (\text{tr} \Sigma^2)^2 - \frac{b}{2} \text{tr} \Sigma^4$$

- Classical domain wall solution:

$$\langle \Sigma \rangle = \begin{cases} 
  V \text{diag}(2,2,-2,-3), & x^5 \to \infty, \\
  -V \text{diag}(2,2,-2,-3), & x^5 \to -\infty, 
\end{cases}$$

- Translational invariance along the extra dimension and SU(5) are simultaneously broken

- GUT relations are easily violated in the 4D viewpoint
Localizing massive X-bosons

- 5D Equations of motion for the SM gauge bosons:

  Wave equation  \[ \text{Wave functions have flat profiles} \]

- 5D Equations of motion for the X-bosons:

  \[
  -\partial_5^2 + M_X^2(x^5)f_X^{(n)}(x^5) = M_X^{(n)}f_X^{(n)}(x^5)
  \]

  \[
  f_X^{(n)}(\xi) \sim (1 - \xi^2)^{(c-n)/2}F_1\left(-n,2c-n+1;c-n+1;\frac{1-\xi}{2}\right)
  \]

  \[
  n = 0, 1, 2, \ldots < c,
  \]

  \[
  \xi = \tanh(mx^5),\ c = \frac{1}{2}\left(\sqrt{1 + \frac{100g_S^2V^2}{m^2}} - 1\right) > 0
  \]

  \[
  M_X^{(n)} = [(2n+1)c - n^2]m^2
  \]

- Zero modes of the X-bosons:

  \[
  f_X^{(0)}(x^5) = \frac{m^{1/2}}{\pi^{1/4}}\sqrt{\frac{\Gamma(c+1/2)}{\Gamma(c)}}\frac{1}{\cosh^c(mx^5)}
  \]

  \[
  M_X^{(0)} = cm^2.
  \]

X-bosons are localized with exponential profiles [c.f. Hamada,Kobayashi,2012]
Proton Stability

- Profiles:

![Graph showing the separation between X-bosons and fermions]

4D $X^{(0)}$-coupling:

$$g_X^{(0)} = g_5 \int_\infty^\infty dx^5 \left[ f_L^{(0)}(x^5) \right]^2 f_X^{(0)}(x^5)$$

- Separation between the X-bosons and fermions. $g_X$ is exponentially suppressed.

- Proton stability.

- No symmetry is imposed to suppress the coupling $g_X$.

[c.f. Hamada, Kobayashi, PTP128, 903 (2012)]
GUT Scale

- Effective X-boson mass:
  
  \[ \frac{M_X^{(0)}}{g_X^{(0)}} > 10^{15} \text{GeV} \]

- Contributions from higher KK X-bosons are also suppressed by a bit larger separation

- Non-observation of exotic colored particles
  
  Inverse of the size of the extra dimension: \( L^{-1} \geq 1\text{TeV} \)

- To suppress proton decay: \( \frac{4}{m} < l < L \)
  
  \[ M_X^{(0)} > 10\text{TeV} \]

- The effects of the X-bosons may be probed at experiments

- \( g_X^{(0)} \) is achieved even with a small X-boson mass

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Mitsuru KAKIZAKI
We proposed a new mechanism to localize X-bosons in extra dimensions.

By separating the X-bosons and fermions, proton decay can be extremely suppressed without imposing symmetries.

The GUT scale can be as low as $O(10)\text{TeV}$.

Future direction: Supersymmetrizing the X-boson localization mechanism.
Backup slides
July 4, 2012:
A new particle was discovered at CERN LHC

- Mass: \( \cong 125 \text{GeV} \)
- Couplings: consistent with the SM Higgs boson ones

The SM is established as a low-energy effective theory

This is not the end of the story
Grand Unified Theories (GUTs)

- The SM gauge interactions are unified:
  \[ SU(3)_C \times SU(2)_L \times U(1)_Y \subset G \]

- The SM fermions (quarks, leptons) are also unified:
  e.g.) \( G = SU(5) \quad (Q, U^C, E^C) = 10 \quad (D^C, L) = 5^* \)

  Charge quantization

- New GUT particles are automatically introduced:
  - X-boson: \( \left( \begin{array}{c} G \\ X \\ W \end{array} \right) \oplus B = 24 \)

  Prediction: Proton is not absolutely stable
Idea of extra dimensions

- **Thread:**
  - **Macroscopic:**
    - 1-dimensional object
  - **Microscopic:**
    - object with internal structure

- **Our world:**
  - **Macroscopic:**
    - 4-dimensional space-time
  - **Microscopic:**
    - higher-dimensional space-time

- **Dispersion relation for a higher dimensional particle:**
  \[ E^2 = p^2 + (p_5^2 + p_6^2 + \cdots + m^2) \]

  **Momentum in the extra dim. = Mass in the 4-d viewpoint**
SM fermion masses and mixings:

\[ m_u : m_c : m_t \sim 10^{-5} : 10^{-3} : 1, \quad \cdots \]
\[ |V_{us}| : |V_{cb}| : |V_{ub}| \sim 0.22 : 0.04 : 0.003 \]

Why are the masses and mixings so hierarchical?

Approaches to this flavor problem:

- Flavor symmetry:
  - U(1) \cite{FroggattNielsen1979}
  - Non-Abelian discrete symmetry \cite{ManyIdeasThisWorkshop}

- Extra dimensions without imposing symmetries:
  - Volume suppression \cite{Yoshioka2000}
  - Localized fields \cite{ArkaniHamedSchmaltz2000}
Four Dimensional Coupling Constants

- 4D coupling constants are given by overlaps of the wave functions
- e.g.) 4D Yukawa interactions:

\[ \mathcal{L}_{4D} = \left( y_{5D} \int dy \, f_{q}^{(0)}(y) f_{u}^{(0)}(y) f_{h}^{(0)}(y) \right) \bar{q}^{(0)}(x) u^{(0)}(x) h^{(0)}(x) \]

Locations of fermion and Higgs fields can account for the fermion mass hierarchy
1. Motivation
2. Localized fields in extra dimensions
3. Breaking of translational invariance by an adjoint Higgs
   - Doublet-Triplet Splitting in SUSY GUTs
     [MK and Masahiro Yamaguchi, Prog. Theor. Phys. 107, 433 (2002)]
   - Fermion mass hierarchy in SUSY GUTs
     [MK and Masahiro Yamaguchi, Int. J. Mod. Phys. A 19, 1715 (2004)]
   - Proton stability in low-scale GUTs
4. Summary
Doublet Triplet Splitting in SUSY GUTs

- 5D superpotential for $H = \begin{pmatrix} H_T \\ H_u \end{pmatrix}$:
  
  $$W_5 = H^C(y) \left[ \partial_y + f\Xi(y) + g\Sigma(y) + M \right] H(y)$$
  
  $\langle \Xi(y) \rangle = 2\xi^2 y : \text{SU(5) singlet kink}$

- SU(5) breaking by $\langle \Sigma \rangle \neq 0$:
  
  $$g\langle \Sigma \rangle + M = \begin{cases} 
  M_T \equiv M + 2gV & \text{for } H_T \\
  M_u \equiv M - 3gV & \text{for } H_u 
  \end{cases}$$

  $l(H_T) \neq l(H_u)$

- Yukawa interaction with a singlet $S$:
  
  $$W_5 = \int dy S(x, y) H(x, y) \bar{H}(x, y) \quad \langle S \rangle \sim 10^{16} \text{GeV}$$

- Exponential profiles
  
  $$M_{T,T} \sim 10^{16} \text{GeV} \quad M_{H_u,H_d} \sim 100 \text{GeV}$$

[See also Maru, PLB522, 117(2001); Haba, Maru, PLB532, 93(2002)]
Fermion mass hierarchy in SUSY GUTs

- SU(5) breaking by $\langle \Sigma \rangle \neq 0$
  splits quarks from leptons in the same multiplets

  - Mass difference between the down-type quarks and charged leptons

- Exponential profiles
  - Realistic pattern of the fermion masses, mixings and CKM phase

[See also Maru, PLB522, 117(2001)]

$\Psi(10) = (Q, U^C, E^C)$

$\Sigma \neq 0$