Finite Temperature Effects in Warm Hybrid Inflation

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Inflationary Cosmology - Two Dynamical Realizations

There are different dynamical realizations of Inflation.

▶ Cold Inflation

▶ The inflaton is treated as an isolated system.
▶ Other initial components of energy density are redshifted away.
▶ A separate reheating phase after inflation brings the universe to a radiation dominated regime.

▶ Warm Inflation

▶ Interactions leading to dissipation of inflaton energy to other degrees of freedom.
▶ Inflationary expansion occurs concurrently with particle production.
▶ Radiation can eventually dominate the energy density without a separate reheating phase.

Warm Inflation

- Warm inflation is realised when a dissipative term, $\Upsilon$, is included as a friction term in the evolution equation for the inflaton

\[
\begin{align*}
\text{Cold Inflation} & : \ddot{\phi}(t) + 3H\dot{\phi}(t) + V_\phi = 0 \\
\text{Warm Inflation} & : \ddot{\phi}(t) + (3H + \Upsilon)\dot{\phi}(t) + V_\phi = 0
\end{align*}
\]

- Energy lost by the inflaton field is gained by some other fluid $\rho_\alpha$
- If $\rho_\alpha = \rho_R$ then the evolution equation for the radiation energy density becomes

\[
\begin{align*}
\text{Cold Inflation} & : \dot{\rho}_R + 4H\rho_R = 0 \\
\text{Warm Inflation} & : \dot{\rho}_R + 4H\rho_R = \Upsilon\dot{\phi}^2
\end{align*}
\]

- Radiation is not necessarily redshifted.

Warm Inflation Model Building - SUSY model

- Supersymmetry protects the potential from large radiative corrections

- Inflaton potential is protected from large thermal corrections*
  - Fields coupled to the inflaton, denoted $\chi$, are heavy because of the coupling to the inflaton
  - Heavy $\chi$ fields are in turn coupled to light $y$ fields

- This can be realised with the superpotential

\[ W = W(\Phi) + g\Phi X^2 + hXY^2 \]

- The scalar component of the superfield $\Phi$ describes the inflaton field $\phi$
- $X$ is the superfield for the heavy catalyst fields $\chi$
- The last term allows the heavy catalyst field to decay into light degrees of freedom in the supermultiplet $Y$

* “Dissipation coefficients from scalar and fermion quantum field interactions”, M Bastero-Gil, A Berera, R Ramos, JCAP 1109:033, 2011
Hybrid Inflation

- This decay mechanism for the inflaton can be readily realised with a hybrid model of inflation
- Inflaton field is responsible for slow-roll inflation
- Waterfall field triggers the end of inflation

- The hybrid inflation potential is

\[
V(\phi, \chi) = \frac{1}{4\lambda}(M^2 - \lambda \chi^2)^2 + \frac{1}{2}\phi^2 + \frac{1}{2}g^2\phi^2\chi^2
\]

- Effective mass squared of the \(\chi\) field is \(-M^2 + g^2\phi^2\)
- When \(\phi > \phi_c = M/g\) the minimum of \(V\) is at \(\chi = 0\)
- When \(\phi < \phi_c = M/g\) the tachyonic instability drives the system to a global minimum at \(\phi = 0\) and \(\chi^2 = M^2/\lambda\)

Supersymmetric Hybrid Inflation

- Supersymmetric hybrid inflation can be realised with the superpotential

\[ W = W(\Phi) + g\Phi(X^2 - M^2) + hXY^2 \]

- The scalar and fermionic components of the $X$ superfields, $\chi = (\chi_R + i\chi_i)/\sqrt{2}$ and $\psi_\chi$, acquire non-vanishing masses during inflation

\[ m_{\chi I}^2 = 2g^2(\phi^2 + M^2) \]
\[ m_{\chi R}^2 = 2g^2(\phi^2 - M^2) \]
\[ m_{\bar{\chi}}^2 = 2g^2\phi^2 \]

- The hybrid transition will happen when $\phi = \phi_c = M$

In warm inflation the coupling between the inflaton, waterfall fields and light fields leads to the dissipation of inflaton energy during inflation.

\[
\begin{align*}
\text{Inflaton} & \quad \rightarrow \quad \text{Waterfall fields} & \quad \rightarrow \quad \text{Light fields} \\
\phi & \quad \rightarrow \quad \chi & \quad \rightarrow \quad y
\end{align*}
\]

The waterfall fields are unstable against decay into the Y sector

\[
\chi \rightarrow yy, \psi_y \psi_y \quad \psi_\chi \rightarrow y\psi_y
\]

Allows inflaton energy to be transferred to Y sector

Dissipation acts as a friction term \(Y \dot{\phi}\) in the equation of motion of the inflaton field \(\phi\)
Dissipative Coefficient

The dissipative coefficient \( \Upsilon \) receives leading contributions from

- **Low-momentum contribution** \( \Upsilon_{lm} \)
  - \( \Upsilon_{lm} \) corresponds to off-shell production
  - \( \Upsilon_{lm} \) dominates for \( m_i >> T \)

- **Pole contribution** \( \Upsilon_{pole} \)
  - \( \Upsilon_{pole} \) corresponds to on-shell production
  - \( \Upsilon_{pole} \) dominates for \( m_i << T \)

The expression for the Dissipative coefficient \( \Upsilon \) is

\[
\Upsilon = \Upsilon_{lm} + \Upsilon_{pole} = \sum_{i = \chi_R, I} \left[ 0.64 h^2 g^8 N_x N_y \frac{T^3 \phi^6}{m_i^8} + \frac{16}{\sqrt{2\pi}} \frac{g^2 N_x}{h^2 N_y} \left( \frac{2g^2 \phi^2}{2g^2 \phi^2 + m_i^2} \right) \sqrt{Tm_i e^{-m_i/T}} \right]
\]

* “General Dissipation coefficient in low-temperature warm inflation”, M Bastero-Gil, A Berera, R Ramos, J Rosa, JCAP 01, 016 (2013)
The Scalar Potential at One Loop

- Interactions between the inflaton $\phi$ and the waterfall field $\chi$ lead to radiative corrections to the scalar potential.

- The scalar potential during inflation is given by the tree-level potential, $V_0 + f(\phi)$, and radiative corrections given by the Coleman-Weinberg potential.*

- At one loop this gives

\[
V(\phi) = V_0 + f(\phi) + \frac{1}{32\pi^2} \sum_{\chi_{R,I},\psi,\chi} m_i^4(\phi) \left[ \log \left( \frac{m_i^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right]
\]

- where $V_0 = g^2 M^4$, $f(\phi) = |W'(\phi)|^2$, $\mu$ is the renormalization scale, and we sum over the $\chi$ mass multiplets

\[
\begin{align*}
m^2_{\chi I}(\phi) &= 2g^2(\phi^2 + M^2) \\
m^2_{\chi R}(\phi) &= 2g^2(\phi^2 - M^2) \\
m^2_{\bar{\chi}}(\phi) &= 2g^2 \phi^2
\end{align*}
\]

A Heat Bath during Inflation

- In warm inflation radiation is not necessarily red-shifted during inflation

\[ W = W(\Phi) + g\Phi(X^2 - M^2) + hXY^2 \]

- The last term in \( W \) allows the waterfall field to decay to light degrees of freedom in the \( Y \) supermultiplet
- This allows for particle creation in the \( Y \) sector and the formation of a thermal bath
- Presence of a thermal bath induces thermal corrections to the masses of the waterfall field \( \chi \) and \( \psi_\chi \) through the following interactions with the light scalars and fermions in the \( Y \) sector, \( y \) and \( \psi_y \)

* “General dissipation coefficient in low-temperature warm inflation”, M Bastero-Gil, A Berera, R Ramos, J Rosa, JCAP 01, 016 (2013)
Thermal Corrections to the Waterfall field

- Allowing for the existence of $g_*$ light degrees of freedom in the thermal bath coupling to fields in the $X$ sector, the waterfall field masses are shifted by a positive factor denoted $\alpha^2 T^2$

$$m_{\chi_I}^2(\phi, T) = 2g^2(\phi^2 + M^2) + \alpha^2 T^2$$

$$m_{\chi_R}^2(\phi, T) = 2g^2(\phi^2 - M^2) + \alpha^2 T^2$$

$$m_{\chi}^2(\phi, T) = 2g^2 \phi^2 + \alpha^2 T^2$$

- If only fields in the $Y$ sector are present in the thermal bath then $g_* = (15/4)N_y$ and $\alpha = h\sqrt{N_y/2}$

- Inserting into the Coleman-Weinberg expression for radiative corrections at one loop the potential becomes

$$V(\phi, T) = V_0 + f(\phi) + \frac{1}{32\pi^2} \sum_{\chi_R, I, \psi_\chi} m_i^4(\phi, T) \left[ \log \left( \frac{m_i^2(\phi, T)}{\mu^2} \right) - \frac{3}{2} \right]$$

Inflation ends when the mass squared of the real scalar component $m_{\chi_R}^2$ becomes negative

Without thermal corrections $m_{\chi_R}^2(\phi) = 2g^2(\phi^2 - M^2)$

Critical value of $\phi$ for the end of inflation, $\phi_c$, is constant

$$\phi_c = M$$

Thermal corrections mean $m_{\chi_R}^2$ becomes temperature dependent

The critical value of $\phi$ now evolves as

$$\phi_c = \left( M - \frac{\alpha^2}{2g^2} T^2 \right)^{1/2}$$
Numerical Results

Evolution of $\phi/M_p$ for initial conditions that give spectral index $n_s = 0.962$ and $r = 0.019$. Solid lines show evolution with thermal corrections included.

**Figure:** Evolution of $\phi/m_p$ for standard cold inflation, warm inflation with $\Upsilon = \Upsilon_{lm}$ and $\alpha^2 = 0$, warm inflation with $\Upsilon = \Upsilon_{lm} + \Upsilon_{pole}$ and $\alpha^2 = 0$, $\Upsilon = \Upsilon_{lm}$ and $\alpha^2 = h^2 N_y/2$, and warm inflation with $\Upsilon = \Upsilon_{lm} + \Upsilon_{pole}$ and $\alpha^2 = h^2 N_y/2$. $h = 0.34$, $M = 0.158 m_p$, $N_x = 5 \times 10^6$, $g = 10^{-3}$, $N_y = 50$, and initial value of $\Phi(0) = 2.58 m_p$. 
Numerical Results

Evolution of $m_{\chi R}/T$ for both low-momentum contribution on its own and the full dissipative coefficient for initial conditions that give spectral index $n_s = 0.962$ and $r = 0.019$. Solid lines show evolution with thermal corrections included.

Figure: Evolution of $m_{\chi R}/T$ for standard cold inflation, warm inflation with $\Upsilon = \Upsilon_{lm}$ and $\alpha^2 = 0$, warm inflation with $\Upsilon = \Upsilon_{lm} + \Upsilon_{pole}$ and $\alpha^2 = 0$, $\Upsilon = \Upsilon_{lm}$ and $\alpha^2 = h^2 N_y/2$, and warm inflation with $\Upsilon = \Upsilon_{lm} + \Upsilon_{pole}$ and $\alpha^2 = h^2 N_y/2$. $h = 0.34$, $M = 0.158m_p$, $N_x = 5 \times 10^6$, $g = 10^{-3}$, $N_y = 50$, and initial value of $\Phi(0) = 2.58m_p$. 
Conclusion

- Dissipation in warm inflation causes a friction term $\Upsilon$ in the inflaton’s equation of motion, leading to particle production occurring concurrently with inflationary expansion.

- The dissipative coefficient receives a contribution from low-momentum “off-shell” modes and from the pole “on-shell” modes of the waterfall field.

- Thermal corrections increase the mass of the waterfall field lowering the critical value of $\phi$, but also suppresses dissipation so the critical value is reached faster.

- With thermal corrections included in the example shown the low-momentum contribution alone increases the number of e-folds by $\sim 10$ e-folds.

- With thermal corrections included in the example shown the pole contribution adds a further $\sim 15$ e-folds taking the total to $\sim 52$ e-folds with spectral index $n_s = 0.962$ and $r = 0.019$. 

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THANK YOU FOR LISTENING
Figure: Evolution of $\eta/(1 + Q)$, $Q$, $T/H$, $\Gamma/H$ and $m_{\chi R}/T$ for warm inflation with $\Upsilon = \Upsilon_{lm} + \Upsilon_{pole}$ and $\alpha^2 = h^2 N_y/2$. $h = 0.34$, $M = 0.158 m_p$, $N_x = 5 \times 10^6$, $g = 10^{-3}$, $N_y = 50$, and initial value of $\Phi(0) = 2.58 m_p$. Spectral index $n_s = 0.962$.