Nucleon decay via dimension-6 operators in anomalous $U(1)_A$ SUSY GUT models and $E_6 \times SU(2)_F$ SUSY GUT model

Yu Muramatsu (Nagoya university)

Collaborator: Nobuhiro Maekawa (Nagoya university, KMI)

based on arXiv:1307.7529 [hep-ph] and more

PASCOS 2013 @ NTU
1. Introduction and previous work

2. $E_6 \times SU(2)_F$ Model
Grand Unified Theory

many advantages and several realistic models

How do we identify models as “the” grand unified theory?

Model identification

\[ \Lambda_{GUT} = 10^{12-16} \text{GeV} \gg \Lambda_{LHC} = 10^{3-4} \text{GeV} \]

It is hard to identify models by accelerator.

In this work, we try GUT model identification by “the nucleon decay” (arXiv:1307.7529 [hep-ph]).

• phenomenon from GUT(bSM)
• expected to be observed in the near future
The nucleon decay in SUSY GUT models

via dimension-6 operators
the main decay mode

\[ p \rightarrow \pi^0 + e^c \]

\( \tau_N \sim 10^{36} \text{ years} \) (proportional to \( \Lambda_{\text{SUSY GUT}}^4 \))

via diemnsion-5 operators
the main decay mode

\[ p \rightarrow K^+ + \nu^c \]

\( \tau_{p \rightarrow \pi^0 + e^c} \geq 1.3 \times 10^{34} \) @ Super-K

\( \Lambda_{\text{SUSY GUT}} \sim 10^{16} \text{ GeV} \)

\( \tau_N \sim 10^{36} \text{ years} \) (proportional to \( \Lambda_{\text{SUSY GUT}}^4 \))

significant interaction

weake suppression
Anomalous $U(1)_A$ SUSY GUT

constructed under “natural assumptions”

• consider all operators which are allowed by symmetries
• consider effects of all higher-order operators
• operators’ coefficients are order 1
Anomalous $U(1)_A$ SUSY GUT models realize many features.

- doublet-triplet splitting
- gauge coupling unification
- realistic quark and lepton masses and mixings etc.

Nucleon decay is one of the most interesting predictions.

**via dimension-6 operators**

$\Lambda_A \sim \lambda^{-a} \Lambda_{SUSY\ GUT} < \Lambda_{SUSY\ GUT}$

GUT scale in anomalous $U(1)_A$ SUSY GUT models

$\Lambda_A \sim \lambda^{0.5} \Lambda_{SUSY\ GUT} = 10^{16}$ GeV

The nucleon decay via dimension-6 operators is significant.

**via dimension-5 operators**

Natural realization of doublet-triplet splitting suppresses the nucleon decay via dimension-5 operators.

We study the nucleon decay via dimension-6 operators.
Diagonalizing Matrix

\[
\psi^L_i Y_{ij} \psi^c_R = (L^\dagger_\psi \psi^L_i) (L^T_\psi Y R^c_\psi)_{ij} (R^\dagger_\psi \psi^c_R)_j \\
= \psi'_L \psi^c_R
\]

\[L_\psi, R_\psi: \text{diagonalizing matrix}\]

In the Standard Model (weak interaction)

\[U_{CKM} = L_u^\dagger L_d, \quad U_{MNS} = L_{\nu}^\dagger L_e\]

In nucleon decay operators (X type gauge int.)

Diagonalizing matrices appear directly.

Experimentally,

We cannot measure diagonalizing matrices directly.
Theoretically,

For example in the minimal SU(5) GUT model,

Yukawa matrices have few degrees of freedom.

Diagonalizing matrices are strongly restricted.

But, \( Y_d = Y_e^T \)

To realize realistic quark and lepton masses and mixings, we introduce additional interactions.

Additional degrees of freedom

Diagonalizing matrices have uncertainties.
In the previous work (arXiv:1307.7529 [hep-ph]), we try to identify GUT models by nucleon decay.

Two important ratios

\[ R_1 \equiv \frac{\Gamma_{n \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}} \]

To identify grand unification group

\[ R_2 \equiv \frac{\Gamma_{p \rightarrow K^0 + \mu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}} \]

To identify Yukawa structure

\[
\mathcal{L}_{\text{eff}} = \frac{g_{\text{GUT}}^2}{M_X^2} \left\{ \frac{1}{2} (e^{c}_{R_i} u_{R_j})(u^{c}_{L_i} d_{L_j}) + \frac{1}{2} (e^{c}_{R_i} u_{R_j})(u^{c}_{L_i} d_{L_j}) \right\} \\
+ \frac{g_{\text{GUT}}^2}{M_{X'}^2} \left\{ (e^{c}_{L_i} u_{L_j})(u^{c}_{R_i} d_{R_j}) - (\nu^{c}_{L_i} d_{L_j})(\nu^{c}_{R_i} d_{R_j}) \right\} \\
+ \frac{g_{\text{GUT}}^2}{M_{X''}^2} \left\{ (e^{c}_{L_i} u_{L_j})(u^{c}_{R_i} D_{R_j}) - (\nu^{c}_{L_i} d_{L_j})(\nu^{c}_{R_i} D_{R_j}) \right\}
\]
\[ R_1 = \frac{\Gamma_{n \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}} \]

\[ R_2 = \frac{\Gamma_{p \rightarrow K^0 + \mu^+ + \mu^-}}{\Gamma_{p \rightarrow \pi^0 + e^c}} \]
1. Introduction and previous work

2. $E_6 \times SU(2)_F$ Model
$E_6 \times SU(2)_F$ Model

Yukawa structure at GUT scale is restricted.

27 dimensional representation for matter : $\Psi$

$\Psi_\alpha (\alpha = 1,2) : SU(2)_F$ doublet for first- and second-generation matters

$\Psi_3 : SU(2)_F$ singlet for third-generation matters

Features

• Realize realistic quark and lepton masses and mixings within restricted Yukawa structure
• Solve SUSY CP problem (Chromo-EDM constraint) by spontaneous CP violation mechanism
• Suppress Flavor Changing Neutral Current processes
in previous work

\[ Y_u = \begin{pmatrix} y_{u11} \lambda^6 & y_{u12} \lambda^5 & y_{u13} \lambda^3 \\ y_{u21} \lambda^5 & y_{u22} \lambda^4 & y_{u23} \lambda^2 \\ y_{u31} \lambda^3 & y_{u32} \lambda^2 & y_{u33} \end{pmatrix} \]

\[ Y_d = \begin{pmatrix} y_{d11} \lambda^6 & y_{d12} \lambda^{5.5} & y_{d13} \lambda^5 \\ y_{d21} \lambda^5 & y_{d22} \lambda^{4.5} & y_{d23} \lambda^4 \\ y_{d31} \lambda^3 & y_{d32} \lambda^{2.5} & y_{d33} \lambda^2 \end{pmatrix} \]

\[ Y_e = \begin{pmatrix} y_{e11} \lambda^6 & y_{e12} \lambda^5 & y_{e13} \lambda^3 \\ y_{e21} \lambda^{5.5} & y_{e22} \lambda^{4.5} & y_{e23} \lambda^{2.5} \\ y_{e31} \lambda^5 & y_{e32} \lambda^4 & y_{e33} \lambda^2 \end{pmatrix} \]

\[ 9 \times 3 = 27 \mathcal{O}(1) \text{ parameters for masses and mixings} \]

in \( E_6 \times SU(2)_F \) Model

\[ Y_u = \begin{pmatrix} 0 & \frac{1}{3} y_{u12} \lambda^5 & 0 \\ -\frac{1}{3} y_{u12} \lambda^5 & y_{u22} \lambda^4 & y_{u23} \lambda^2 \\ 0 & y_{u23} \lambda^2 & y_{u33} \end{pmatrix} \]

\[ Y_d = \begin{pmatrix} y_{d11} \lambda^6 & y_{d12} \lambda^{5.5} & \frac{1}{3} y_{d13} \lambda^5 \\ y_{d21} \lambda^5 & y_{d22} \lambda^{4.5} & y_{d23} \lambda^4 \\ y_{d31} \lambda^3 & y_{d32} \lambda^{2.5} & y_{d33} \lambda^2 \end{pmatrix} \]

\[ Y_e = \begin{pmatrix} y_{d11} \lambda^6 & y_{e12} \lambda^5 & 0 \\ 0 & y_{e22} \lambda^{4.5} & y_{e32} \lambda^2 \\ -y_{e12} \lambda^5 & y_{d23} \lambda^4 & y_{d33} \lambda^2 \end{pmatrix} \]

\[ 4 + 9 + 3 = 16 \mathcal{O}(1) \text{ parameters for masses and mixings} \]
\( \theta_{ij}^{\psi_{L,R}} \): mixing angle for diagonalizing matrix of \( \psi_{L,R} \)

\[
L_\psi, R_\psi \equiv \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23}^{\psi_{L,R}} & s_{23}^{\psi_{L,R}} \\
0 & -s_{23}^{\psi_{L,R}} & c_{23}^{\psi_{L,R}}
\end{pmatrix} \begin{pmatrix}
c_{13}^{\psi_{L,R}} & 0 & s_{13}^{\psi_{L,R}} \\
0 & 1 & 0 \\
-s_{13}^{\psi_{L,R}} & 0 & c_{13}^{\psi_{L,R}}
\end{pmatrix} \begin{pmatrix}
c_{12}^{\psi_{L,R}} & s_{12}^{\psi_{L,R}} & 0 \\
-s_{12}^{\psi_{L,R}} & c_{12}^{\psi_{L,R}} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\( s_{ij}^{\psi_{L,R}} \sim \theta_{ij}^{\psi_{L,R}}, c_{ij}^{\psi_{L,R}} \sim 1 \) (main order)

\[
\theta_{13}^{uL} = 0 \quad \theta_{13}^{uR} = 0 \quad \theta_{13}^{eL} = 0
\]

\[
\theta_{23}^{uL} = \theta_{23}^{uR} \quad \theta_{23}^{dL} = \theta_{23}^{eR} \quad \theta_{12}^{eR} = \theta_{23}^{eL} \theta_{12}^{eL}
\]

\[
|\theta_{12}^{uL}| = |\theta_{12}^{uR}| = \sqrt{m_u/m_c} \quad \frac{m_\mu}{m_\tau} = -\frac{\theta_{13}^{eR}}{\theta_{13}^{eL}}
\]

\[
\theta_{13}^{eL} \theta_{13}^{eR} m_\tau - \theta_{13}^{dL} \theta_{13}^{dR} m_b + \theta_{12}^{eL} \theta_{12}^{eR} m_\mu - \theta_{12}^{dL} \theta_{12}^{dR} m_s = 1
\]

\( m_d - m_e \)

\( m_b = m_\tau \)

11 conditions @ GUT scale
Yukawa matrix 1-loop renormalization group equation

\[
\frac{d}{dt} Y_{u,d,e} = \frac{1}{16\pi^2} \beta^{(1)}_{Y_{u,d,e}}
\]

\[
\begin{align*}
\beta^{(1)}_{Y_u} &= Y_u \left\{ 3 \text{Tr}(Y_u Y_u^\dagger) + 3 Y_u^\dagger Y_u + Y_d^\dagger Y_d - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2 \right\} \\
\beta^{(1)}_{Y_d} &= Y_d \left\{ \text{Tr}(3 Y_d Y_d^\dagger + Y_e Y_e^\dagger) + 3 Y_d^\dagger Y_d + Y_u^\dagger Y_u - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^2 \right\} \\
\beta^{(1)}_{Y_e} &= Y_e \left\{ \text{Tr}(3 Y_d Y_d^\dagger + Y_e Y_e^\dagger) + 3 Y_e^\dagger Y_e - 3 g_2^2 - \frac{9}{5} g_1^2 \right\}
\end{align*}
\]

\[
\begin{align*}
\theta_{13}^{uL} &= \text{negligible} & \theta_{13}^{uR} &= \text{negligible} & \theta_{13}^{eL} &= \text{negligible} \\
\theta_{23}^{uL} &= \theta_{23}^{uR} & \theta_{23}^{dL} &= \theta_{23}^{eR} & \theta_{12}^{eR} &= \theta_{23}^{eL} \theta_{12}^{eL} \\
|\theta_{12}^{uL}| &= |\theta_{12}^{uR}| = \sqrt{m_u/m_c} & m_\mu &= - \frac{\theta_{13}^{eR}}{\theta_{13}^{eL}} & m_\tau & m_\tau & m_b & m_b + \theta_{12}^{eL} \theta_{12}^{eR} m_\mu & \theta_{12}^{dL} \theta_{12}^{dR} m_s &= 1 \\
\theta_{13}^{eL} \theta_{13}^{eR} m_\tau - \theta_{13}^{dL} \theta_{13}^{dR} m_b + \theta_{12}^{eL} \theta_{12}^{eR} m_\mu & \theta_{12}^{dL} \theta_{12}^{dR} m_s &= 1 \\
\end{align*}
\]

9 conditions for mixing angles @ low energy scale
Parameters for real diagonalizing matrix:
3 parameters (mixing angle) for each matrix

7 diagonalizing matrices: $L_u, L_d, L_e, L_v, R_u, R_d, R_e$

$\Rightarrow$ 21 parameters

To realize
\[ U_{CKM} = L_u^\dagger L_d, \quad U_{MNS} = L_v^\dagger L_e \]
we use 6 parameters.

$E_6$ GUT model in previous work:
21 – 6 = 15 parameters

In this work we use 9 conditions.
$E_6 \times SU(2)_F$ GUT model:
21 – 6 - 9 = 6 parameters (+ 1 sign)
$R_1 = \frac{\Gamma_{n \rightarrow \pi^0 + \nu^e}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$

$R_2 = \frac{\Gamma_{p \rightarrow K^0 + \mu^+ + \nu}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$

- $E_6$ model 1
- $E_6 \times SU(2)_F$ model 1

10^6 model points

preliminary
In previous $E_6$ model

\[ L_{e12} \sim \lambda^{0.5} \sim 0.5, \quad L_{e13} \sim \lambda^3 \sim 0.01 \]

In $E_6 \times SU(2)_F$ model

\[ L_{e12} \sim 0.39 < \lambda^{0.5}, L_{e13} = 0 \]

Small mixing from first-generation charged lepton

\[ \tau_{p \rightarrow \pi^0 + e^+} \geq 1.3 \times 10^{34} \text{ years} \]

@Super-K

\[ \tau_{p \rightarrow \pi^0 + e^+} \geq 1.3 \times 10^{35} \text{ years} \]

@Hyper-K (planning)
Summary

• We calculate nucleon lifetimes. Especially, we pay attention to uncertainties of diagonalizing matrices.

• Flavor symmetry restrict Yukawa structure and diagonalizing matrices, therefore we can reduce uncertainties of diagonalizing matrices.

• In many model points $R_1$ and $R_2$ tend to be smaller than these in the $E_6$ model of previous work by conditions for mixing angles.
Thank you for your attention.
Back up slide
Why do we use $n \to \pi^0 + \nu^c$? not $p \to \pi^+ + \nu^c$?

- sensitivity

\[ \frac{\Gamma_{n\to\pi^0+\nu^c}}{\Gamma_{p\to\pi^0+e^c}} \]

- form factor

\[ \langle \pi^0 | (ud)_{\Gamma} u_{\Gamma'} | p \rangle = \langle \pi^0 | (du)_{\Gamma} d_{\Gamma'} | n \rangle \]

We can cancel form factor and uncertainty of that!!
In the minimal SO(10) GUT model, all SM matters and $\nu_R$ in each generation belong to only 16 rep.

Diagonalizing matrices for each matter are same at the GUT scale.

It is hard to realize realistic fermion masses and mixings in the minimal SO(10) GUT model.

**SO(10) GUT with 10 rep.**

\[
\begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix}
\]

diagonalizing matrix for each matter that belong to 16 rep. in SO(10).

\[
\begin{pmatrix}
1 & \lambda & \lambda^3 \\
\lambda & 1 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix}
\]

diagonalizing matrix for each matter that belong to 10 rep. in SU(5).

**CKM type**

**MNS type**

\[
\begin{pmatrix}
1 & \lambda^{0.5} & \lambda \\
\lambda^{0.5} & 1 & \lambda^{0.5} \\
\lambda & \lambda^{0.5} & 1
\end{pmatrix}
\]

diagonalizing matrix for each matter that belong to $\bar{5}$ rep. in SU(5).
dependence for anti-electron mode

\[
\begin{pmatrix}
A_{11} & A_{12}\lambda \\
A_{21}\lambda & A_{22}
\end{pmatrix}
\]

\(A_{ij} \) : order 1

uncertainty

large off diagonal element

unitary condition

small diagonal element

= large mixing

Lifetime of \( p \rightarrow \pi^0 + e^c, K^0 + \mu^c \) mode become long.

Lifetime of \( p \rightarrow \pi^0 + \mu^c, K^0 + e^c \) mode become short.
dependence for anti-neutrino mode

\[ L_u, R_e, L_e, L_d, R_d \]

\[ R_u = 1_{3 \times 3} \]
suppression of dim 5 effective int.

mass matrix of triple Higgs

\[
\begin{pmatrix}
3_H & 3_{H'}
\end{pmatrix}
\begin{pmatrix}
0 & m \\
m & M
\end{pmatrix}
\begin{pmatrix}
\overline{3}_H \\
\overline{3}_{H'}
\end{pmatrix}
\]

effective colored Higgs mass

\[
m^\text{eff}_c \sim \frac{m^2}{M}
\]

In anomalous $U(1)_A$ SUSY GUT models, we can realize $m^\text{eff}_c > 10^{18}\text{GeV}$ strongly suppressed
lower limit of nucleon lifetime

- $p \rightarrow e^+ \pi^0$
  - just reaching to $10^{34}$ yrs
- $p, n \rightarrow (\pi^+ \text{ or } \mu^+) + (\pi, \eta, p, \omega)$
  - many modes updated
- SUSY favored $p \rightarrow \nu K^+ 3.3 \times 10^{33}$ yrs
- $K^0$ modes, $\nu \pi 0, \nu \pi^+$ to be updated
- Super-K has searched only for favored modes and got most stringent limits of $O(10^{32})-O(10^{34})$ years
- It is important to test many decay modes
  - radiative decays
    - $p \rightarrow (\pi^+, \mu^+) + \gamma$
  - invisible decays
    - $n \rightarrow \nu \nu \nu$
  - neutron-antineutron oscillation
  - di-nucleon decay ($|\Delta B| = 2$)
    - $pp \rightarrow ?$
    - $pn \rightarrow ?$
    - $nn \rightarrow ?$
future lower limit of nucleon lifetime @ Hyper-Kamiokande (10 years running)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Sensitivity (90% CL)</th>
<th>Current limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p \to e^+\pi^0$</td>
<td>$13 \times 10^{34}$ years</td>
<td>$1.3 \times 10^{34}$ years</td>
</tr>
<tr>
<td>$p \to \mu^+\pi^0$</td>
<td>$9.0 \times 10^{34}$</td>
<td>$1.1 \times 10^{34}$</td>
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<tr>
<td>$p \to e^+\eta^0$</td>
<td>$5.0 \times 10^{34}$</td>
<td>$0.42 \times 10^{34}$</td>
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<td>$p \to \mu^+\eta^0$</td>
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<td>$p \to e^+\rho^0$</td>
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<td>$p \to \mu^+\rho^0$</td>
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<td>$0.02 \times 10^{34}$</td>
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<td>$p \to \mu^+\omega^0$</td>
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<td>$0.08 \times 10^{34}$</td>
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<tr>
<td>$n \to e^+\pi^-$</td>
<td>$3.8 \times 10^{34}$</td>
<td>$0.20 \times 10^{34}$</td>
</tr>
<tr>
<td>$n \to \mu^+\pi^-$</td>
<td>$2.9 \times 10^{34}$</td>
<td>$0.10 \times 10^{34}$</td>
</tr>
<tr>
<td>$p \to \bar{\nu}K^+$</td>
<td>$2.5 \times 10^{34}$</td>
<td>$0.40 \times 10^{34}$</td>
</tr>
</tbody>
</table>

arXiv:1109.3262 [hep-ex]
\[ Y_u = \begin{pmatrix}
0 & \frac{1}{3} y_{u12} \lambda^5 & 0 \\
-\frac{1}{3} y_{u12} \lambda^5 & y_{u22} \lambda^4 & y_{u23} \lambda^2 \\
0 & y_{u23} \lambda^2 & y_{u33}
\end{pmatrix} \]

origin of \( \frac{1}{3} \)

\[ \Psi_a A \Psi_a H \]

\( H \): fundamental representation Higgs

\( A \): adjoint Higgs \( \langle A \rangle \propto Q_{B-L} \)

\( \frac{1}{3} \) is B-L charge of quarks.