LARGE volume scenario in 5D SUGRA

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Introduction

Low scale SUSY is a possible candidate for new physics. e.g. dark matter, gauge hierarchy, etc...

The order parameter of the SUSY breaking is gravitino mass \( m_{3/2} \)

\[
    m_{3/2} = \langle e^{K/2} W \rangle
\]

\[
    W = W_0 + \cdots
\]

If the constant term is forbidden only by the global R symmetry, the constant term can arise from the gravitational effects!

To realize low scale SUSY breaking, we have to tune the constant term \( W_0 \)
Introduction

LARGE volume scenario (LVS) in string theory

Moduli stabilization: exponentially large extra dimension

SUSY breaking: The scale is much smaller than the Planck scale

Without fine-tuned small constant term!

V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo (2005)
J. P. Conlon, F. Quevedo, and K. Suruliz (2005)
LVS was constructed in the string theory.

Our question: Can we realize the LVS in a simple set-up?

In this talk, I will show

- Realization of the LVS in effective theory of 5D SUGRA model
- Possible mass spectrum in 5D LVS
General 5D SUGRA on $S^1/Z_2$ → General 4D effective theory reduction

T. Kugo and K. Ohashi (2001)

5D vector multiplet → 4D vector multiplet

5D vector multiplet → 4D chiral multiplet

* Only the vector or the chiral multiplet has its zero-mode.

Zero mode of the chiral multiplet = moduli multiplet
Multiple moduli in 5D SUGRA

General 5D SUGRA on $S^1/Z_2$ → General 4D effective theory reduction

T. Kugo and K. Ohashi (2001)

5D vector multiplet

4D vector multiplet
4D chiral multiplet = moduli multiplet

Well known set-up: one modulus
= radion
Multiple moduli in 5D SUGRA

General 5D SUGRA on $S^1/Z_2$

T. Kugo and K. Ohashi (2001)

5D vector multiplet

4D vector multiplet

4D chiral multiplet = moduli multiplet

General set-up: multiple moduli

= radion + non-geometric moduli

The norm function and Kahler potential in 5D SUGRA

The size of the extra dim. \( L_{\text{phys}} = \langle \mathcal{N}^{1/3} \rangle \)

\[ \mathcal{N} = C_{IJK} \text{Re}T^I \text{Re}T^J \text{Re}T^K \]
The norm function and Kahler potential in 5D SUGRA

The size of the extra dim. \[ \mathcal{L}_{\text{phys}} = \langle \mathcal{N}^{\frac{1}{3}} \rangle \]

\[ \mathcal{N} = C_{IJK} \text{Re} T^I \text{Re} T^J \text{Re} T^K \]

Kähler potential: \[ K = - \log \mathcal{N} \]

\[ K_I K^{IJ} K_{\bar{J}} = 3 \]  
(no-scale relation)
The norm function and Kahler potential in 5D SUGRA

The size of the extra dim. $L_{\text{phys}} = \langle \mathcal{N}^{\frac{1}{3}} \rangle$

$\mathcal{N} = C_{IJK} \text{Re}T^I \text{Re}T^J \text{Re}T^K$

1-loop corrected Kahler potential: $K = -\log(\mathcal{N} + \xi)$

$K_I K^{IJ} K_{\bar{J}} = 3 + \frac{6\xi}{\hat{\mathcal{N}}} + \cdots$
LVS in 5D SUGRA

\[ K = - \log(\mathcal{N} + \xi) \quad W = W_0 + A e^{-a T_s} \]

where

\[ a = \mathcal{O}(4\pi^2) \]
\[ W_0 = \mathcal{O}(M_{pl}^3) \]

\[ \mathcal{N} = (\text{Re} T_b)^3 - C_s (\text{Re} T_s)^3 \]

\[ T_b = \tau_b + i \rho \quad \sim \text{radion} \]
\[ T_s = \tau_s + i \sigma \quad \sim \text{non-geometric modulus} \]
Moduli stabilization

\[ V \sim \frac{1}{N} \left( \frac{2N}{3C_s \tau_s} (aA)^2 e^{-2a\tau_s} + 4a\tau_s W_0 A e^{-a\tau_s} \cos(a\sigma) \right) + \frac{6\xi W_0^2}{N^2} \]

\[ = \frac{2(aA)^2}{3C_s \tau_s} e^{-2a\tau_s} + 4a\tau_s W_0 A \cos(a\sigma) \frac{e^{-a\tau_s}}{N} + 6\xi W_0^2 \frac{1}{N^2} \]

Non-perturbative term \sim Volume suppressed term

\[ \langle N \rangle \sim \frac{3\xi W_0 e^{a\langle \tau_s \rangle}}{a\langle \tau_s \rangle A} \quad \langle \tau_s \rangle \sim \left( \frac{\xi}{C_s} \right)^{\frac{1}{3}} \]

\[ L_{\text{phys}} = \langle N^{\frac{1}{3}} \rangle \gg 1 \quad \text{(In Planck unit)} \]

Exponentially large extra dimension!
To realize the almost vanishing cosmological constant, an extra SUSY breaking effect is needed.

We assume the SUSY breaking sector $X$

$$\langle V_{\text{AdS}} \rangle = -\frac{27\xi W_0^2}{2(a\langle \tau_s \rangle)^2\mathcal{N}^2} \quad \Rightarrow \quad |F^X|^2 = K_{XX}^{-1} |\langle V_{\text{AdS}} \rangle|$$

The relative size of the F-term differ from the Kahler metric of $X$

$$|F^X| \sim \begin{cases} 
\frac{m_{3/2}}{(\log \mathcal{N})\mathcal{N}^{1/2}} & (X \text{ lives in the bulk}) \\
\frac{m_{3/2}}{(\log \mathcal{N})\mathcal{N}^{1/3}} & (X \text{ lives on a brane})
\end{cases}$$
SUSY breaking in 5D LVS

Gravitino mass: $m_{3/2} = e^K \langle W \rangle \sim \frac{W_0}{\sqrt{N}} \sim \mathcal{O}(M_{pl}/\sqrt{N}) \ll M_{pl}$

F-terms:

$$\frac{F^{T_b}}{T_b + \bar{T}_b} \sim \frac{W_0}{\sqrt{N}} = m_{3/2}$$

$$\frac{F^{T_s}}{T_s + \bar{T}_s} \sim \frac{W_0}{(a \tau_s) \sqrt{N}} \sim \frac{m_{3/2}}{\log N}$$

$$\frac{F^X}{M_{pl}} \sim \begin{cases} \mathcal{O} \left( \frac{m_{3/2}}{(\log N) N^{1/3}} \right) & \text{(on Brane)} \\ \mathcal{O} \left( \frac{m_{3/2}}{(\log N) N^{1/2}} \right) & \text{(in the bulk)} \end{cases}$$

Small SUSY breaking scale can be realized naturally!
Anomaly mediation is much suppressed by the leading no-scale structure.

\[ \frac{F^\phi}{\phi} = \frac{m_3/2}{\mathcal{N}} \ll m_3/2 \]

The 4D effective action of the bulk fields

\[ \int d^4 \theta |\phi|^2 \Omega + \int d^2 \theta f^{(r)} \mathcal{W}_\alpha \mathcal{W}^\alpha + \text{h.c.} + \cdots \]

where

\[ \Omega = \cdots + 2N^{1/3} Y_a (T_b, T_s) |Q_a|^2 + \cdots \]

\[ f^r = C_s^r T_s \]

Scalar mass

\[ m_a \sim m_{3/2} \]

Gaugino mass

\[ M_r \sim \frac{m_{3/2}}{\log \mathcal{N}} \]

This spectrum is similar to ones of the pure gravity mediation and mini-split SUSY.

M. Ibe and T. T. Yanagida (2011)

We assume the following couplings between brane localized sector and $X$

\[
\int d^4 \theta |\phi|^2 \Omega_b + \int d^2 \theta f_b^{(R)} W_{\alpha} W^{\alpha} + \text{h.c.} + \cdots
\]

\[
\Omega_b = h_A \left( 1 - \frac{\zeta(3)}{8\pi^2 N} \right) |q^A|^2 - \kappa_{AX} |q^A|^2 |X|^2
\]

\[
f_b^{(R)} = f_0^R + k_X X
\]

Scalar mass

\[
m_A \sim \begin{cases} 
\frac{m_{3/2}}{\mathcal{N}^{1/2}} \\
\frac{m_{3/2}}{(\log \mathcal{N}) \mathcal{N}^{1/3}}
\end{cases}
\]

Gaugino mass

\[
M_R \sim \begin{cases} 
\frac{m_{3/2}}{(\log \mathcal{N}) \mathcal{N}^{1/2}} \\
\frac{m_{3/2}}{(\log \mathcal{N}) \mathcal{N}^{1/3}}
\end{cases}
\]

($X$ in the bulk)

($X$ on the brane)
We assume the following couplings between brane localized sector and $X$:

\[ \int d^4 \theta |\phi|^2 \Omega_b + \int d^2 \theta f_b^{(R)} \mathcal{W}_\alpha \mathcal{W}^\alpha + \text{h.c.} + \cdots \]

\[ \Omega_b = h_A \left(1 - \frac{\zeta(3)}{8\pi^2 \mathcal{N}}\right) \]

\[ f_b^{(R)} = f_0^{(R)} + k_X X \]

- **Scalar mass**
  \[ m_A \sim \begin{cases} \frac{m_{3/2}}{\mathcal{N}^{1/2}} & \text{(in the bulk)} \\ \frac{m_{3/2}}{(\log \mathcal{N})\mathcal{N}^{1/3}} & \text{(on the brane)} \end{cases} \]

- **Gaugino mass**
  \[ M_R \sim \begin{cases} \frac{m_{3/2}}{(\log \mathcal{N})\mathcal{N}^{1/2}} & \text{(in the bulk)} \\ \frac{m_{3/2}}{(\log \mathcal{N})\mathcal{N}^{1/3}} & \text{(on the brane)} \end{cases} \]
Summary

We construct the LVS in 5D SUGRA and show the patterns of mass spectrum in this model.

• General set-up of 5D SUGRA → multi-moduli
• Casimir term → breaking the no-scale relation

• Mass spectrum of the brane matters depends on the SUSY breaking sector
• Mini-splitting between the scalars and gauginno mass is realized in some cases
Thank you.
Appendix
The value of $\xi$

$$\xi \equiv \frac{(\bar{n}_H - n_V - 1)\zeta(3)}{32\pi^2}$$

$n_V$ : The number of the vector multiplets

$\bar{n}_H$ : The effective number of the hypermultiplets

where

$$\bar{n}_H = \sum_{\alpha} n_{\alpha} \frac{Z(d_{\alpha} \cdot \text{Re}T/2)}{Z(0)}$$

$$Z(x) = -\int_0^\infty d\lambda \lambda \ln\left(2e^{-\sqrt{\lambda^2 + x^2}} \sinh \sqrt{\lambda^2 + x^2}\right)$$
# Multiplets in 4D effective theory

<table>
<thead>
<tr>
<th>4D multiplet</th>
<th>5D vector multiplet (even)</th>
<th>5D vector multiplet (odd)</th>
<th>Hypermultiplet</th>
</tr>
</thead>
<tbody>
<tr>
<td>parity</td>
<td>$V^I$ Vector</td>
<td>$\tilde{T}^I$ chiral</td>
<td>$Q_a$ chiral</td>
</tr>
<tr>
<td>Zero mode</td>
<td>$V^I$</td>
<td>$\tilde{V}^{I'}$ vector</td>
<td>$Q_a$ chiral</td>
</tr>
<tr>
<td>Role in 4D</td>
<td>vector (gauge)</td>
<td>$T^{I'}$ chiral</td>
<td>$Q'_a$ chiral</td>
</tr>
</tbody>
</table>

- **4D multiplet**:
  - $V^I$: Vector
  - $\tilde{T}^I$: Chiral

- **5D vector multiplet (even)**:
  - $V^I$: Vector

- **5D vector multiplet (odd)**:
  - $\tilde{V}^{I'}$: Vector
  - $T^{I'}$: Chiral

- **Hypermultiplet**:
  - $Q_a$: Chiral
  - $Q'_a$: Chiral

- **Role in 4D**:
  - **vector (gauge)**: moduli
  - **matter**:
KK mass & moduli masses

Gravitino mass: \[ m_{3/2} = \frac{W_0}{\sqrt{N}} \sim \mathcal{O}(M_{pl}/\sqrt{N}) \]

KK mass & Moduli mass: \[ m_{\tau_b} \sim \frac{m_{3/2}}{\sqrt{N}} \quad m_\rho \sim 0 \]
\[ m_{\tau_s} \sim m_\sigma \sim (\log N) m_{3/2} \quad m_{KK} \sim \frac{M_{pl}}{N^{1/3}} \]

\[ M_{pl} \gg m_{KK} \gg m_{3/2} \]

Analysis by the effective theory is valid!