Higgs Vacuum Stability and Physics
Beyond the Standard Model

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Combined 2012-13 data: It is a Higgs boson, maybe even the Higgs boson

CMS Preliminary $m_H = 125.7$ GeV

$\mu = 0.65$

ATLAS Preliminary

$W,Z H \rightarrow bb$

$\mu = 1.15 \pm 0.62$

$H \rightarrow \tau\tau$

$\mu = 1.10 \pm 0.41$

$H \rightarrow \gamma\gamma$

$\mu = 0.77 \pm 0.27$

$H \rightarrow WW$

$\mu = 0.68 \pm 0.20$

$H \rightarrow ZZ$

$\mu = 0.92 \pm 0.28$

$\sqrt{s} = 7$ TeV, $L \leq 5.1$ fb$^{-1}$ $\sqrt{s} = 8$ TeV, $L \leq 19.6$ fb$^{-1}$

$\mu = 1.30 \pm 0.20$
The **Standard Model Higgs with $m_h=125-126$ GeV**

- **Naturalness problem**: somewhat heavy than typical prediction of the supersymmetric models and somewhat light than typical prediction of technicolour models.

- More notably, the Standard Model vacuum state $|0\rangle_{EW}$

  $$EW \langle 0 | h | 0 \rangle_{EW} = v_{EW} \approx 246 \text{ GeV}$$

is a false (local) vacuum. The true vacuum state

$$\langle 0 | h | 0 \rangle \sim M_P \approx 10^{18} \text{ GeV},$$

and it carries large negative energy density $\sim - (M_P)^4$.

- **How long does the electroweak vacuum live?**
**EW vacuum lifetime: flat spacetime estimate**

- Electroweak Higgs doublet (in the unitary gauge): \( H = \begin{pmatrix} 0 \\ h(x)/\sqrt{2} \end{pmatrix} \)

\[
V_{H}^{(0)}(h) = \frac{\lambda}{8} \left( h^2 - v_{EW} \right)^2
\]

- Effective (quantum-corrected) potential

\[
V_{H}^{(1-loop)}(h) = \frac{\lambda(h)}{8} \left( h^2 - v_{EW} \right)^2,
\]

\[
\lambda(h) = \lambda(\mu) + \beta_\lambda \ln(h/\mu)
\]

\[
(4\pi)^2 \beta_\lambda = -6y_t^4 + 24\lambda^2 + \ldots
\]

\[
y_t(m_t) \approx 1, \quad \lambda(m_h) \approx 0.13 \quad \rightarrow \beta_\lambda < 0
\]
EW vacuum lifetime: flat spacetime estimate

Figure 1: Two loop running of the Higgs quartic coupling in the SM.

(a) $m_h = 125 \text{ GeV}$

(b) $m_h = 126 \text{ GeV}$

Instability scale $\lambda(\mu_i) = 0, \mu_i \approx 10^{10} \text{ GeV}$

AK & A. Spencer-Smith, arXiv:1305.7283
**EW vacuum lifetime: flat spacetime estimate**

- Large field limit:
  \[ V_H = \frac{\beta_\lambda \ln(h/\mu_i)}{4} h^4, \quad \beta_\lambda = \beta_\lambda|_{\mu=\mu_i} \]
  \[ h_* = \mu_i e^{-1/4} \]

- Using Coleman’s prescription, one can calculate that the decay of electroweak vacuum is dominated by small size Lee-Wick bounce solution,
  \[ R \sim 1/\mu_m \approx 10^{-17}/\text{GeV}, \quad \beta_\lambda|_{\mu=\mu_m} = 0 \]
  \[ S_{\text{LW}} = \frac{8\pi^2}{3|\lambda(\mu_m)|}, \quad |\lambda(\mu_m)| \approx 0.01 - 0.02 \]
  \[ P_{\text{EW}} = e^{-p} \approx 1, \quad p = (\mu_m/H_0)^4 \exp(-S_{\text{LW}}) \ll 1 \]

**Electroweak vacuum in the Standard Model is metastable!**
**EW vacuum in inflationary universe**

- Electroweak vacuum decay may qualitatively differ in cosmological spacetimes:
  1. Thermal activation of a decay process, $T_r < \mu_i$
  2. Production of large amplitude Higgs perturbations during inflation, $H_{\text{inf}} < \mu_i$  


  The bound that follows from the above consideration can be avoided, e.g., in curvaton models, or when $m_h^{\text{eff}} > H_{\text{inf}}$

- Actually, the dominant decay processes are due to instantons, (Hawking-Moss, or more generic CdL) [AK & A. Spencer-Smith, Phys Lett B 722 (2013) 130 [arXiv:1301.2846]]

\[
V(h, \phi) = V_H(h) + V_{\text{inf}}(\phi) + V_{H-\text{inf}} \\
V_{\text{inf}}(\phi) = V_{\text{inf}} + V^*(\phi - \phi_{\text{inf}}) + 1/2V''(\phi - \phi_{\text{inf}})^2 + ... \\
\epsilon = \frac{M^2_P}{2} \left( \frac{V^*}{V_{\text{inf}}} \right)^2 << 1, \quad -1 << \eta = M^2_P \frac{V''}{V_{\text{inf}}} << 1
\]
**EW vacuum in inflationary universe**

- Fixed background approximation: \( \phi = \phi_{\text{inf}}, \ d{s}^2 = d\chi^2 + \rho^2(\chi)d\Omega_3, \)

\[
\rho(\chi) = H_{\text{inf}}^{-1} \sin(H_{\text{inf}}\chi), \ \chi = t^2 + r^2, \ \chi \in [0, \pi/H_{\text{inf}}], \ H_{\text{inf}}^2 = V_{\text{inf}}/3M_P^2
\]

- EoM for Higgs field:

\[
\ddot{h} + 3H_{\text{inf}} \cot(H_{\text{inf}}\chi)\dot{h} = \frac{\partial V(\phi_{\text{inf}}, h)}{\partial h}
\]

\[
\dot{h}(0) = \dot{h}(\pi/H_{\text{inf}}) = 0
\]

\[
h_L(x_*) = h_R(x_*)
\]

*Fig. 1. The Higgs potential. For large values of the Higgs field \( h \), the electroweak vacuum configuration is regarded as trivial, \( v_{\text{EW}} \approx 0 \).*
EW vacuum in inflationary universe

- Hawking-Moss instanton:  
  \[
  \frac{\partial V}{\partial h} = 0, \quad h(x) = h_*,
  \]

  \[p \approx \exp \left\{ -\frac{8\pi^2}{3} \frac{V_H(h_*) + V_{H-\inf}(\phi_{\inf}, h_*)}{H_{\inf}^4} \right\}\]

- For \(V_{H-\inf}(\phi_{\inf}, h_*) << V(h_*)\), HM transition generates a fast decay of the electroweak vacuum, unless

  \[H_{\inf} < 10^9 (10^{12}) \text{ GeV}\]

  \[m_h = 126 \text{ GeV}, \quad m_t = 174(172) \text{ GeV}\]

- Together with \(n_s < 1\), this implies that only small-field inflationary models are allowed with a negligible tensor/scalar:

  \[r < 10^{-11} (10^{-5})\]
EW vacuum in inflationary universe

- Consider, 
  \[ V_{H-\text{inf}} = \frac{\alpha}{2} h^2 \phi^2 \quad (\alpha > 0), \quad m_h^{\text{eff}} = \alpha^{1/2} \phi_{\text{inf}} > H_{\text{inf}} \]

[similar consideration applies \( \frac{\xi}{2} R^2 h^2 \) ]

\[ h_* = \left( -\frac{\alpha}{\lambda} \right)^{1/2} \phi_{\text{inf}} > \mu_i, \quad (\lambda(h_*) < 0) \]

- Large-field chaotic inflation \[ V_{\text{inf}} = 1/2 m_\phi^2 \phi^2, \quad m_\phi = 10^{-5} M_P, \] with

\[ \alpha > 1.4 \sqrt{|\lambda|} \left( H_{\text{inf}} / \phi_{\text{inf}} \right)^2 > 6 \cdot 10^{-12} . \]

- Naturalness constraint:

\[ \alpha < 64 \pi^2 \left( m_\phi / m_h \right)^2 \approx 2 \cdot 10^{-20} \]

Tuning is needed!
EW vacuum in inflationary universe

- In the limit \( m_h^{\text{eff}} >> H_{\text{inf}} \)

\[
h'' + 3h' \chi = \frac{\partial V(\phi_{\text{inf}}, h)}{\partial h}, \quad [x = m_h^{\text{eff}} \chi]
\]

\[
h(x) = \begin{cases} 
8h_R \left( 8 + \left( \frac{h_R}{h_*} \right)^2 x^2 \right)^{-1}, & 0 \leq x < x_* \\
\frac{x_* h_*}{x(J_1(ix_*)) + iY_1(-ix_*)} \left( J_1(ix) + iY_1(-ix) \right), & x_* < x < \infty
\end{cases}
\]

\[
x_* = \frac{2\sqrt{2}h_*}{h_R} \left( \frac{h_R}{h_*} - 1 \right)^{1/2}
\]

\[
B_{\text{CdL}} = -\frac{2\pi^2}{\lambda} I < 0, \quad I = \int_0^\infty x^3 dx \left[ h^2(x) \left( 1 - \frac{h^2(x)}{2h_*^2} \right) \right] < 0, \quad \lambda(\mu > \mu_i) < 0.
\]

\[
p \propto \exp\{ -B_{\text{CdL}} \} >> 1 \quad \text{EW vacuum is unstable!}
\]
EW vacuum in inflationary universe

- Fast decay of EW ceases inflation globally (no eternal inflation)
  \[ e^{3H_{\text{inf}} \tau} e^{-\left(\tau H_{\text{inf}}\right)^4 p} \]
  \[ \tau_{\text{stop}} \approx \left(\frac{3}{p}\right)^{1/3} H_{\text{inf}}^{-1} < 1.4H_{\text{inf}}^{-1} \]
- The above considerations applies to models with curvaton

Conclusions:

- For the ‘unperturbed’ SM potential the condition of vacuum metastability rules out all large scale models. All models with sizeable Higgs-inflaton interactions are ruled out.
- Observation of tensor perturbations in the CMB by the Planck satellite would provide a strong indication of new physics beyond the Standard Model.
Neutrino masses and vacuum stability


\[ \beta^{(1)}_\lambda = 24\lambda^2 - 6y_t^4 + \frac{3}{4}g_2^4 + \frac{3}{8} \left( g_1^2 + g_2^2 \right)^2 + \lambda \left( -9g_2^2 - 3g_1^2 + 12y_t^2 \right) \]

1. Extension of the electroweak gauge sector
2. Extension of a scalar sector
3. Extension of the fermionic sector

- Working with MS-bar couplings:
  1. Modification of beta-functions above the particle mass threshold
  2. Finite threshold correction due to the matching of low and high energy theories

- Neutrino oscillations (= masses) provide the most compelling evidence for the physics beyond the Standard Model.
Neutrino masses and vacuum stability


- Type I see-saw models: additional massive sterile neutrinos – not capable to solve the vacuum stability problem

- Type III see-saw models: additional electroweak-triplet fermions – may solve the problem for very specific range of parameters

More promising candidates:

- Type III see-saw: additional electroweak-triplet scalar

- Left-right symmetric models: additional gauge bosons, scalars and fermions
Type II see-saw models and vacuum stability

Scalar potential:

\[
V(\phi, \Delta) = -m_\phi^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + m_\Delta^2 \text{tr}(\Delta^\dagger \Delta) + \frac{\lambda_1}{2} (\text{tr}(\Delta^\dagger \Delta))^2 \\
+ \frac{\lambda_2}{2} [\text{tr}(\Delta^\dagger \Delta)^2 - \text{tr}(\Delta^\dagger \Delta)^2] + \lambda_4 (\phi^\dagger \phi) \text{tr}(\Delta^\dagger \Delta) + \lambda_5 \phi^\dagger [\Delta^\dagger, \Delta] \phi + \left[ \frac{\lambda_6}{\sqrt{2}} \phi^T i \sigma_2 \Delta^\dagger \phi + h.c. \right]
\]

Neutrino masses:

\[
\frac{1}{\sqrt{2}} (y_\Delta)_{fg} l^T_L f C i \sigma_2 \Delta l^g_L + h.c.
\]
Type II see-saw models and vacuum stability

(a) $m_{\Delta} = 1 \text{ TeV}, \lambda_6 = 0.01m_{\Delta}$

(b) $m_{\Delta} = 100 \text{ TeV}, \lambda_6 = 0.13m_{\Delta}$.

(c) $m_{\Delta} = 10^8 \text{ GeV}, \lambda_6 = 0.17m_{\Delta}$.

(d) $m_{\Delta} = 100 \text{ TeV}, \lambda_6 = 0.57m_{\Delta}$.

Figure 2: One loop running of the Higgs quartic coupling in the type-II seesaw model, with $m_h = 125 \text{ GeV}$ and $m_t = 173 \text{ GeV}$. 

P. T.
Type II see-saw models and vacuum stability

(a) $m_\Delta = 10^8\text{GeV}$, $\lambda_6 = 0.17m_\Delta$.

(b) $m_\Delta = 1\text{ TeV}$, $\lambda_6 = 0.01m_\Delta$.

Figure 3: Conditions for stability and absence of tachyonic modes in the scalar potential of the type-II seesaw model, with $m_h = 125$ GeV and $m_t = 175$ GeV. Line labels correspond to the stability conditions (3.9)-(3.16) with $a \equiv \lambda$, $b \equiv \lambda_1$, $c \equiv \lambda_1 + \frac{\lambda_2}{2}$, $d/e \equiv \lambda_4 \pm \lambda_5 + 2\sqrt{\lambda\lambda_1}$, $f/g \equiv \lambda_4 \pm \lambda_5 + 2\sqrt{\lambda \left(\lambda_1 + \frac{\lambda_2}{2}\right)}$, $h \equiv \lambda_6$, $i \equiv -\lambda_5v_\Delta$, $j \equiv -2\lambda_5v_\Delta - \frac{\lambda_2v_\Delta^3}{v_{EW}}$. 
Alternating LR-symmetric model with universal seesaw for all fermion masses

Scalar sector:

\[ \phi_L \in (1, 1, 2, 1/2), \quad \phi_R \in (1, 2, 1, 1/2). \]

\[ V(\phi_L, \phi_R) = -m^2 \left( \phi_L^\dagger \phi_L + \phi_R^\dagger \phi_R \right) + \frac{\lambda}{2} \left( \phi_L^\dagger \phi_L + \phi_R^\dagger \phi_R \right)^2 + \sigma \phi_L^\dagger \phi_L \phi_R^\dagger \phi_R. \]

New fermions:

\[ N_L, N_R \in (1, 1, 1, 0), \]
\[ E_L, E_R \in (1, 1, 1, -1), \]
\[ U_{iL}, U_{iR} \in (3, 1, 1, 2/3), \]
\[ D_{iL}, D_{iR} \in (3, 1, 1, -1/3), \]
Alternating LR-symmetric model with universal seesaw for all fermion masses

(a) Running gauge couplings.  (b) Running Yukawa couplings.

(c) Running Higgs quartic coupling

Figure 4: Running couplings in the ALRSM with $M_T = 4.7 \times 10^9$ GeV and $m = 1 \times 10^{10}$ GeV.
Conclusions:

- Electroweak vacuum (in)stability may already provide a hint for physics beyond the Standard Model.

- Interesting interplay with cosmological inflation: observation of gravitational waves will provide a strong hint for a new physics that is responsible for the stabilization of the vacuum.

- The most promising models of neutrino masses in this regard are: type II see-saw and LR models. They may potentially tested at the LHC.