The Higgs boson mass in a natural MSSM with nonuniversal gaugino masses

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with
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A Standard Model-like Higgs particle has been discovered.

\[ m_{Higgs} = 126.0 \pm 0.4 \pm 0.4 \text{[GeV]} \text{ (ATLAS)} \]

\[ m_{Higgs} = 125.3 \pm 0.4 \pm 0.5 \text{[GeV]} \text{ (CMS)} \]
Fine-tuning problem in the Standard Model

\[ m_{Higgs}^2 = m_{bare}^2 - \Lambda_{CUT}^2 + \cdots \]

If the cut-off scale is of order GUT scale,

\[(125 \, [\text{GeV}])^2 \simeq (10^{16} \, [\text{GeV}])^2 - (10^{16} \, [\text{GeV}])^2\]

In the MSSM,

\[ m_{Higgs}^2 = m_{bare}^2 + (\Lambda_{CUT}^2 - \Lambda_{CUT}^2) + m_{soft}^2 \log \left( \frac{\Lambda_{CUT}}{m_{soft}} \right) + \cdots \]

One of motivations for SUSY; Solution of the fine-tuning problem.
Fine-tuning problem in the Standard Model

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One of motivations for SUSY; Solution of the fine-tuning problem.
SUSY little hierarchy problem in the MSSM
(Fine-tuning problem for EWSB)

\[
\frac{1}{2} m_Z^2 = -|\mu|^2 - m_{H_u}^2 + \cdots \quad (\tan \beta \gg 1)
\]

\[m_Z \cdots \text{Z boson mass} \quad \mu \cdots \text{higgsino mass}\]

\[m_{H_u} \cdots \text{SUSY breaking soft term of } H_u\]

\[\tan \beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}\]

We need \(\mu \sim m_{H_u} \sim O(m_Z)\) to avoid the tuning problem.
We examined the possibility to realize $\sim 125$ GeV Higgs and avoid the SUSY little hierarchy problem.
In the MSSM, lightest CP-even Higgs boson mass is evaluated as \[1\]

\[
m_h^2 \simeq m_Z^2 \cos^2(2\beta) \text{ at the tree level}
\]

\[
m_h^2 \simeq m_Z^2 \cos^2(2\beta) \left( 1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} t \right) + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[ \frac{1}{2} X_t + t + \frac{1}{16\pi^2} \left( \frac{3}{2} \frac{m_t^2}{v^2} - 32\pi\alpha_3 \right) \right] (X_t t + t^2)
\]

There are two possibilities to realize \( \sim 125 \) GeV Higgs.

1. **High-scale SUSY breaking**\( (m_\tilde{t} > 10 \, \text{TeV}) \)

\[
t \equiv \ln(m_t^2/m_\tilde{t}^2)
\]

\( m_t \cdots \text{top mass} \)

\( m_{\tilde{t}} \cdots \text{left handed stop mass} \)

\( m_{\tilde{t}_3} \cdots \text{right handed stop mass} \)

\[
\delta m_{H_u}^2 \sim -\frac{3y_t^2}{4\pi^2} m_t^2 \ln\frac{\Lambda_{\text{CUT}}}{m_\tilde{t}}
\]

2. **Large stop mixing**

\[
r_a = \frac{|\tilde{A}_t|}{m_\tilde{t}} \simeq \sqrt{6} \quad \Rightarrow \text{Higgs mass is maximally enhanced.}
\]
In the MSSM, lightest CP-even Higgs boson mass is evaluated as\[1\]

$$m_h^2 \simeq m_Z^2 \cos^2(2\beta) \text{ at the tree level}$$

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There are two possibilities to realize $\sim 125$ GeV Higgs.

① High-scale SUSY breaking ($\tilde{m}_t > 10 \text{ [TeV]}$)

\[ m^2_{Q_3} \ldots \text{left handed stop mass} \]
\[ m^2_{U_3} \ldots \text{right handed stop mass} \]

\[ \delta m^2_{H_u} \sim -\frac{3y_t^2}{4\pi^2 m_t^2} \ln \frac{\Lambda_{\text{cut}}}{m_{\tilde{t}}} \]

\[ t \equiv \ln \left(\frac{m_t^2}{\bar{m}_t^2}\right) \]
\[ \bar{m}_t \ldots \text{top mass} \]

\[ \text{-------> SUSY little hierarchy problem} \]

② Large stop mixing

\[ r_a = \frac{|\tilde{A}_t|}{m_{\tilde{t}}} \simeq \sqrt{6} \]

Higgs mass is maximaly enhanced.
In the MSSM, lightest CP-even Higgs boson mass is evaluated as [1]

\[ m_h^2 \simeq m_Z^2 \cos^2(2\beta) \] at the tree level

\[ m_h^2 \simeq m_Z^2 \cos^2(2\beta) \left( 1 - \frac{3}{8\pi^2} \frac{\overline{m}_t^2}{v^2} t \right) + \frac{3}{4\pi^2} \frac{\overline{m}_t^4}{v^2} \left[ \frac{1}{2} X_t + t + \frac{1}{16\pi^2} \left( \frac{3}{2} \frac{\overline{m}_t^2}{v^2} - 32\pi\alpha_3 \right) (X_t t + t^2) \right] \]

There are two possibilities to realize \( \sim 125 \text{ GeV} \) Higgs.

① High-sale SUSY breaking (\( m_{\tilde{t}} > 10 \text{ [TeV]} \))

\[ t \equiv \ln\left( \frac{m_t^2}{\overline{m}_t^2} \right) \]

\( \overline{m}_t \cdots \) top mass

\[ m_{Q_3}^2 \cdots \) left handed stop mass

\[ m_{U_3}^2 \cdots \) right handed stop mass

\[ \delta m_{H_u}^2 \sim \frac{3y_t^2}{4\pi^2} m_t^2 \ln \frac{\Lambda_{\text{CUT}}}{m_t} \]

\[ \tilde{A}_t \equiv A_t(m_Z) - \mu(m_Z) \cot \beta \]

\[ m_{\tilde{t}}^2 \equiv \sqrt{m_{U_3}(m_Z)m_{Q_3}(m_Z)} \]

② Large stop mixing

\[ r_a = \frac{|\tilde{A}_t|}{m_\tilde{t}} \simeq \sqrt{6} \quad \text{is large.} \quad \text{Higgs mass is maximally enhanced.} \]
Large stop mixing

How do we get the large stop mixing?

After running a one-loop RG eq. from the GUT scale to the EW scale,

\[ r_a = \frac{|\tilde{A}_t|}{m_{\tilde{t}}} \approx \sqrt{6} \]

\[ \tilde{A}_t(m_Z) \approx -0.04M_1(m_{GUT}) - 0.21M_2(m_{GUT}) - 1.90M_3(m_{GUT}) + \cdots \]

\[ m_{Q_3}^2(m_Z) \approx -0.02M_1^2(m_{GUT}) + 0.38M_2^2(m_{GUT}) + 5.63M_3^2(m_{GUT}) + \cdots \]

\[ m_{U_3}^2(m_Z) \approx 0.07M_1^2(m_{GUT}) - 0.21M_2^2(m_{GUT}) + 4.61M_3^2(m_{GUT}) + \cdots \]

**Ratios of the gaugino masses**

\[ r_1 \equiv \frac{M_1(m_{GUT})}{M_3(m_{GUT})}, \quad r_2 \equiv \frac{M_2(m_{GUT})}{M_3(m_{GUT})} \]

If we take \( r_2 \approx \sqrt{\frac{4.6}{0.2}} \approx 4.8 \)

\[ m_{U_3}^2(m_Z) < m_{Q_3}^2(m_Z), \quad A_t^2(m_Z) \]

\[ r_a > 1 \]

Stop mixing is enhanced due to the RG effects.  \[ \implies \text{Higgs mass is enhanced.} \]
Large stop mixing

Solution of the SUSY little hierarchy problem.

After running a one-loop RG eq. from the GUT scale to the EW scale,

\[ \frac{1}{2} m_Z^2 = -|\mu|^2 - m_{H_u}^2 + \cdots \]

Gaugino mass ratio is almost same as the large stop mixing.

\[
M_1 \cdot \text{Bino mass} \\
M_2 \cdot \text{Wino mass} \\
M_3 \cdot \text{Gluino mass}
\]

\[
m_{H_u}^2 (m_Z) \approx 0.17 M_2^2 (m_{\text{GUT}}) - 3.09 M_3^2 (m_{\text{GUT}}) + \cdots
\]

\[
r_2 \equiv \frac{M_2}{M_3} \approx \sqrt{\frac{3.09}{0.17}} \approx 4.3
\]

\[\Rightarrow \delta m_{H_u}^2 (m_Z) \text{ is suppressed.}\]

SUSY little hierarchy problem is relaxed due to the RG effects.

Gaugino mass ratio is almost same as the large stop mixing.
Higgs mass and tuning

There are the parameter spaces which realize the 125 GeV Higgs and avoid the tuning problem.

\[
\Delta_\mu = \frac{|\mu|}{2m_Z^2} \left| \frac{\partial m_Z^2}{\partial |\mu|} \right|, \quad \frac{100}{|\Delta_\mu|} (\%) 
\]

**Input parameter at the GUT scale**

\[
M_3 = 385 \text{[GeV]}, \tan \beta = 15, \\
(3\text{rd. generation}, H_u, H_d) = 200\text{[GeV]}, A_t = -400\text{[GeV]} \\
(1\text{st. and 2 nd. generation}) = 1500\text{[GeV]} 
\]
In Summary, LSP is Higgsino.
Wino is heavier than gluino.
Stop is relatively light.

Reference point: \((r_1, r_2) = (13, 5.4)\)

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\[
100 \times |\Delta\mu^{-1}| \text{ (\%)}
\]

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There are two interesting candidates to realize the nonuniversal gaugino masses at the GUT scale.

i) Moduli mixing (in the higher-dimensional theory)

ii) Mirage mediation

Moduli mediation + Anomaly mediation

Nonuniversal gaugino masses
Summary
Nonuniversal gaugino masses at the GUT scale

We can realize $\sim 125$ GeV Higgs and avoid the SUSY little hierarchy problem at the same time.
Appendix
\[
\frac{dm_Q^2}{dt} \approx -\frac{1}{4\pi^2} \left( \frac{8}{3}g_3^2|M_3|^2 + \frac{3}{2}g_2^2|M_2|^2 + \frac{1}{30}g_1^2|M_1|^2 - \frac{1}{20}g_1^2S \right) \hat{1} \\
+ \frac{1}{8\pi^2} \left( \frac{1}{2}y^u(y^u)^\dagger m_Q^2 + \frac{1}{2}m_Q^2y^u(y^u)^\dagger + y^u m_U^2(y^u)^\dagger + (m_{H_u}^2)y^u(y^u)^\dagger + A^u(A^u)^\dagger \right)
\]

\[
\frac{dm_U^2}{dt} = -\frac{1}{4\pi^2} \left( \frac{8}{3}g_3^2|M_3|^2 + \frac{8}{15}g_1^2|M_1|^2 + \frac{1}{5}g_1^2S \right) \hat{1} \\
+ \frac{1}{4\pi^2} \left( \frac{1}{2}(y^u)^\dagger y^u m_U^2 + \frac{1}{2}m_U^2(y^u)^\dagger y^u + (y^u)^\dagger m_Q^2 y^u + (m_{H_u}^2)(y^u)^\dagger y^u + (A^u)^\dagger A^u \right),
\]

\[
\frac{dm_{H_u}^2}{dt} = -\frac{1}{4\pi^2} \left( \frac{3}{2}g_2^2|M_2|^2 + \frac{3}{10}g_1^2|M_1|^2 - \frac{3}{20}g_1^2S \right) \\
+ \frac{3}{8\pi^2} \text{tr} \left\{ y^u m_Q^2(y^u)^\dagger + y^u m_U^2(y^u)^\dagger + m_{H_u}^2 y^u(y^u)^\dagger + A^u(A^u)^\dagger \right\}, \quad S = m_{H_u}^2 - m_{H_d}^2 + \text{tr} \left( m_Q^2 - m_L^2 - 2m_U^2 + m_D^2 + m_E^2 \right),
\]

Casmir Invariant, gauge couplings and the value of Yukawa coupling determine the ratio of the gaugino masses.
\[
m_{Q_3}^2(m_Z) \simeq -0.02M_1^2 + 0.38M_2^2 - 0.02M_1M_3 - 0.07M_2M_3 + 5.63M_3^2
\]
\[
\quad + (0.02M_2 + 0.09M_3 - 0.02A_t)A_t
\]
\[
\quad - 0.14m_{H_u}^2 + 0.86m_{Q_3}^2 - 0.14m_{U_3}^2
\]

\[
m_{U_3}^2(m_Z) \simeq 0.07M_1^2 - 0.01M_1M_2 - 0.21M_2^2 - 0.03M_1M_3 - 0.14M_2M_3 + 4.61M_3^2
\]
\[
\quad + (0.01M_1 + 0.04M_2 + 0.18M_3 - 0.05A_t)A_t
\]
\[
\quad - 0.27m_{H_u}^2 - 0.27m_{Q_3}^2 + 0.73m_{U_3}^2
\]

\[
A_t(m_Z) \simeq -0.04M_1 - 0.21M_2 - 1.90M_3 + 0.18A_t
\]

\[
m_{H_u}^2(m_Z) \simeq -0.01M_1M_2 + 0.17M_2^2 - 0.05M_1M_3 - 0.20M_2M_3 - 3.09M_3^2
\]
\[
\quad + (0.02M_1 + 0.06M_2 + 0.27M_3 - 0.07A_t)A_t
\]
\[
\quad + 0.59m_{H_u}^2 - 0.41m_{Q_3}^2 - 0.41m_{U_3}^2.
\]
CCA breaking minima

CCA breaking minima exist unless the following condition is satisfied.

\[ |A_t|^2 \leq 3 \left( m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + |\mu|^2 \right) \]

In the case of \( r_a \sim \sqrt{6} \) is dangerous with satisfying \( m_{Q_3} = m_{U_3} \)

In our parameter region, \( |A_t(m_Z)| \sim m_{Q_3}(m_Z) \)

and \( 0 < m_{U_3}(m_Z) < m_{Q_3}(m_Z) \)

This inequality is guaranteed.