Multi-lognormal magnetic field limits from the cosmic microwave background

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Abstract

Primordial magnetic fields (PMF) affect the cosmic microwave background (CMB). We set spectra of PMFs as a multi lognormal distribution (MLND) at the constant variance \( \sigma_{\text{LND}} = 1 \), and constrain the parameters of the strengths of the magnetic field \( B_k \) each characteristic wavenumber \( k \), along with standard cosmological parameters in the flat Universe from the CMB and foreground sources with the Markov Chain Monte Carlo method.
Background

[Observed cosmological magnetic field]

Faraday rotation measure (Hercules, Perseus-Pisces, Abell 2255)
(Clarke et al 2001; Xu et al. 2006,Govoni et al 2006)

\[ B \approx 0.1 \mu G \sim 1.0 \mu G \]

○ The conductivity is very large \(\rightarrow\) the magnetic field is frozen-in the baryon

\[ [\text{magnetic strength}] \propto [\text{density of baryon (}\rho_b\text{)]}^{2/3} \]

Baryon density in clusters of galaxies
Average of the baryon density in Universe \(\sim 10^2 \sim 10^3\)

Magnetic field in cluster of galaxies
Average of magnetic field in Universe

\[ = \left( \frac{\text{baryon density in clusters of galaxies}}{\text{Average of the baryon density in Universe}} \right)^{2/3} \sim 10^2 \]

\[ [\text{PMF}] = B_0(a=1) = 1 \text{ nG} - 10 \text{ nG} \]

The existence of a magnetic field of order \(\text{nG}\) whose field lines collapse as structure forms is one possible explanation for such magnetic fields to galactic clusters.
1. **The Lorentz force** changes vectors of baryons.

2. Vector of photons is changed by **Thomson scattering**. (photons and baryons are **tight-coupled** before the last scattering surface).
Primordial Magnetic Field from Log-Normal Distribution (LND)

Yamazaki, Ichiki and Takahashi, Accepted for publication as Phys. Rev. D (2013)

**PMF from the inflation**

**Power low**: $P(k) \propto k$

PMF generated by after inflation mechanism

If the magnetic fields which have a characteristic scale at cosmological scales exist, it is possible that such fields can be constrained by observation, and if observed it would give us useful information about generation mechanism of cosmological magnetic fields. Hence, in this work, as a toy example we use a log-normal distribution (LND) for the PMF spectrum.

$$f_{\text{LND}}(k; k_{[\text{PMF}]}, \sigma_{\text{LND}}) = \frac{1}{k_{[\text{PMF}]}} \exp \left\{ - \frac{[\ln (k) - \ln (k_{[\text{PMF}])]}{2 \sigma_{\text{LND}}^2} \right\}$$

$$P_{\text{PMF}}(k) = B_{\text{LND}}^2 \frac{(2\pi)^2}{4} \frac{1}{k_{[\text{PMF}]}} \exp \left\{ - \frac{[\ln (k) - \ln (k_{[\text{PMF}])]}{2 \sigma_{\text{LND}}^2} \right\}$$

$$\times \left\{ \int_{k_-}^{k_+} \frac{dk}{\sigma_{\text{LND}} \sqrt{2\pi}} \exp \left[ - \frac{(\ln |k| - \ln |k_{[\text{PMF}]}|)^2}{2 \sigma_{\text{LND}}^2} \right] \right\}^{-1}$$

$B_{\text{LND}}$: magnetic strength in 1 Mpc, $k_{[\text{PMF}]}$: characteristic scale, $\sigma_{\text{LND}}$: scale parameter.
The margins of errors of the observation on $300 < \ell < 1700$ are comparable in the differences between the theoretical CMB with the LND-PMF and the primary ones (without the LND-PMF).
The margins of errors of the observation on $300 < \ell < 1700$ are comparable in the differences between the theoretical CMB with the LND-PMF and the primary ones (without the LND-PMF). The LND-PMF is mainly constrained by the observational data on $300 < \ell < 1700$. 
Results II: The CMB BB mode with the LND-PMF

For higher $l$, the CMB with the PMF effects is better than without the PMF.
Dependence of LND-PMF on $k_{[\text{PMF}]}$

LND-PMFs with $k_{[\text{PMF}]} = 10^{-3} - 10^{-2}$ dominate on higher $\ell$
Result I: Constrained parameters of LND-PMFs by CMB

We obtain constrained PMF strengths by our MCMC analysis

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<th>Parameter</th>
<th>mean</th>
<th>best fit</th>
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<tr>
<td>$</td>
<td>B_{k=10^{-1}}</td>
<td>$ (nG)</td>
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<tr>
<td>$</td>
<td>B_{k=10^{-2}}</td>
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<td>B_{k=10^{-4}}</td>
<td>$ (nG)</td>
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<td>$</td>
<td>B_{k=10^{-5}}</td>
<td>$ (nG)</td>
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</table>

The lower bound of PMF at $k = 10^{-3}$ Mpc$^{-1}$ seems to be limited.
The PMF strength at $k = 10^{-3}$ Mpc$^{-1}$ is accepted by CMB observations?
Result II: Constrained parameters of LND-PMFs by BB mode

- The lower bound of the PMF strength at $k = 10^{-3} \text{ Mpc}^{-1}$ disappears without BB mode.
- The non-zero value of PMF strength is favored by the BB mode data.
We also find that the statistically-significant result \( B_{k=10^{-3}} \neq 0 \) is obtained by the several finite observed data points of the QUAD on the BB mode of the CMB.

**BUT,**

the current BB mode observations would have relatively large systematic and observational errors.  
We need more precise observations of BB modes by future projects for detecting PMF.
We also find that the statistically-significant result \(\mathbf{B}_{k=10^{-3}} \neq 0\) is obtained by the several finite observed data points of the QUAD on the BB mode of the CMB. BUT, the current BB mode observations would have relatively large systematic and observational errors. We need more precise observations of BB modes by future projects for detecting PMF.
**Summary**

○ The PMF effects are sensitive to CMB for higher $l$

We obtain the limited strengths of LND-PMFs as follows;

- $|B_{k=10^{-1}}| (\text{nG}) < 2.921 (68\% \text{CL}), < 4.685 (95\% \text{CL})$ 1.300
- $|B_{k=10^{-2}}| (\text{nG}) < 1.257 (68\% \text{CL}), < 2.090 (95\% \text{CL})$ 0.410
- $|B_{k=10^{-3}}| (\text{nG}) = 6.179 \pm 1.312$ 6.728
- $|B_{k=10^{-4}}| (\text{nG}) < 3.253 (68\% \text{CL}), < 5.310 (95\% \text{CL})$ 0.465
- $|B_{k=10^{-5}}| (\text{nG}) < 6.992 (68\% \text{CL}), < 10.91 (95\% \text{CL})$ 1.766

○ We also find that the statistically-significant result `$B_{k=10^{-3}} \neq 0$" is obtained by the several finite observed data points of the QUAD on the BB mode of the CMB.

○ However, the current BB mode observations would have relatively large errors.

● We need more precise observations of BB modes by future projects for detecting PMF.
the LND-PMF strength at $k = 10^{-3}$ Mpc$^{-1}$ has the small degeneracies with, $A_s$, $\Omega_b h^2$, $\theta$ and $A_{\text{lens}}$. 

Appendix: Degeneracies

$B_{k=10^{-3}}$ [nG]
Results II: The CMB TT mode with the LND-PMF

The margins of errors of the observation on $300 < \ell < 1700$ are comparable in the differences between the theoretical CMB with the LND-PMF and the primary ones (without the LND-PMF). The LND-PMF is mainly constrained by the observational data on $300 < \ell < 1700$. 