Heavy Quark Expansion in Beauty Decays

Nikolai Uraltsev

INFN, Sezione di Milano, Italy
and
PNPI Gatchina, St. Petersburg, Russia
and
Department of Physics, University of Notre Dame
\[
\sin 2\phi_1 = \sin 2\beta = 0.685 \pm 0.032 \pm \text{sys}
\]

A 3\% accuracy in the CP asymmetry – this sets a benchmark for precision

Theoretical accuracy: utilizes a peculiar mechanism for interference, and is based on CP symmetry of strong interaction where \( B^0 \) decays into a CP eigenstate

Bigi, Carter, Sanda, 1981

Practical implementation: yield space oscillations

Azimov, N.U., Khoze 1985

The advance of the Heavy Quark Expansion of QCD allowed to achieve, in some important instances, a comparable or better precision in the dynamic treatment of strong interaction in \( B \) decays, not based on global symmetries
• We have the QCD-based theory of $B$ decays

• It works at the nonperturbative level
  impressive (at times) agreement with experiment
  gives nontrivial predictions
  allows precision extraction of $|V_{cb}|$ and $|V_{ub}|$
  makes suggestions for next generation experiments

• Goal: extending the range of rigorously tractable cases

• Further theory development required, including
  constructing the consistent OPE in $Q \to q$

• We can derive nontrivial predictions for properties
  of individual heavy flavor hadrons

  Inclusive semileptonic $b \to c \ell \nu$ distributions can/should be
  further explored

  More experimental data are needed to
  confirm theory / resolve legacy apparent contradictions

  HQ sum rules, inequalities and their saturation play an
  important role

We have good control over inclusive $B \to X_s + \gamma$ and $B \to X_u \ell \nu$
decay characteristics with the today’s experimental capabilities if
use the full power of the OPE, in particular utilizing OPE fit
results from $B \to X_c \ell \nu$
\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} (G^a_{\mu\nu})^2 + \sum_q \bar{q} (i\mathcal{D} - m_q) q \]

\[ m_q \ll \Lambda_{\text{QCD}} \quad \text{– light quark} \quad u, d, s \]

\[ m_q \gg \Lambda_{\text{QCD}} \quad \text{– heavy quark} \quad Q \quad c, b, t \]

**Heavy quarks:**

High demand for precision, from the first principles of QCD

Heavy quark is static inside a heavy hadron
A source of static Coulomb field for light degrees of freedom \((q, \bar{q}, g)\)
This ‘potential’ is \(Q\)-spin independent

This provides the **Starter Kit** based on heavy quark symmetry

The real **Heavy Quark Theory** comes with dynamics
Lifetimes and inclusive decay widths

\[ \Gamma = \frac{G_F^2 m_b^5}{192\pi^3} \cdot z(m_c, m_b) \cdot N_c \cdot \left(1 - \frac{\alpha_s}{\pi} \ldots \right) \cdot \ldots \begin{cases} |V_{cb}|^2 \\ |V_{ub}|^2 \end{cases} \]

No \( \Lambda_{\text{QCD}}/m_b \) corrections to inclusive widths of heavy flavor hadrons

Bigi, Uraltsev, Vainshtein 1992

Applies to all types: semileptonic, nonleptonic, \( b \rightarrow s + \gamma \), \( b \rightarrow s \ell^+\ell^- \), ...

\[ \frac{\Delta M}{M} \sim \frac{\Lambda_{\text{QCD}}}{m_b} \quad \text{yet} \quad \frac{\Delta \Gamma}{\Gamma} \sim \frac{\Lambda_{\text{QCD}}^2}{m_b^2} \]

\[ M_B = m_b + \overline{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_b} + \frac{\mu_3^2}{m_b^2} + \ldots \]

\( \overline{\Lambda} \) does not affect the width!

Exclusive property of QCD. Follows from the gauge nature of QCD interaction

Exact cancellation of the bound state effects with the final state interaction
Bound state & hadronization effects are given by local HQ operators $\bar{b}b$

Order $1/m_b^2$: $\mu_\pi^2 = \langle B|\bar{b}(i\tilde{D})^2b|B\rangle$, $\mu_G^2 = \langle B|\bar{b}\frac{i}{2}\sigma Gb|B\rangle$

Order $1/m_b^3$: $\rho_D^3 \propto \langle B|\bar{b}\Gamma b\bar{q}\Gamma q|B\rangle$, $\rho_{LS}^3 \propto \langle B|\bar{\sigma} \cdot \vec{\pi} \times \vec{E}|B\rangle$

etc.

Application: the theory of

**Lifetimes of Beauty Hadrons**

BSUV 1992

$$\delta \mathcal{T}_{H_b} \sim \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{m_b^2}\right) + \mathcal{O}\left(\frac{\Lambda^3_{QCD}}{m_b^3}\right) + \ldots$$

- $1/m_b$: No effects
- $1/m_b^2$: $-\frac{1}{2} \frac{\mu_\pi^2}{m_b^2} - c_G \frac{\mu_G^2}{2m_b^2}$ mesons vs. baryons
- $1/m_b^3$: $\langle B|\bar{b}\Gamma q \cdot \bar{q}\Gamma'b|B\rangle$ $B^+$ vs. $B^0$ vs. $B_s$ ...

Weak Annihilation Pauli Interference Weak Scattering

mesons baryons

Bilić, Guberina, Trampetić 1984
Shifman, Voloshin 1985
\[
\frac{\tau_{B^-}}{\tau_{B^0}} \approx 1.05 \quad \text{BU 1992} \quad 1.076 \pm 0.008 \quad \text{exp}
\]

\[
\left| \frac{\tau_{B_s}}{\tau_{B^0}} - 1 \right| \approx 0.02 \quad \text{BU 1992} \quad 0.92 \pm 0.03 \quad \text{exp}
\]

\[
\frac{\tau_{\Lambda_b}}{\tau_{B^0}} \approx 0.9 \quad 0.806 \pm 0.047 \quad \text{exp}
\]

Dynamic accuracy of a few %, with no symmetry to rely on

\[\text{BR}_{sl}\]  \[\text{BR}_{sl}\] vs. \[n_{\text{charm}}\]

\[\text{BR}_{sl} \approx 10.7\%\], possibly on the lower side for theory

Deserves fresh scrutiny. Now theory must be able to calculate more accurately both \[\text{BR}_{sl}\] and \[\langle n_c \rangle\] separately, modulo reliability of the \[b \to c \bar{c}s\] channel

Proper treating \[m_c\] is important, this yields larger \[\langle n_c \rangle\] confirmed by BaBar

Semileptonic decays offer richer phenomenology and better theoretical environment
Semileptonic decays

Practical applications: Extracting $|V_{cb}|$, $|V_{ub}|$ from $\Gamma_{sl}(B)$

Need accurate values of the QCD parameters $m_b$, $m_c$, $\mu_\pi^2$, $\mu_G^2$, $\rho_D^3$, ...

Replace models and their attributes used early on $m_b$, $m_c$, $\mu_\pi^2$, ... (properly defined) can be determined from the semileptonic $(b\to s+\gamma)$ decay distributions themselves

A robust analysis is required without relying on $1/m_c$ expansion

Now is adopted for experimental analyses

The special role of the hadronic mass moments:

if $m_c$ were large enough, the first would yield $\Lambda$, the second $\mu_\pi^2$, the third $\rho_D^3$ more or less directly

Experiment provides many observables, e.g. $\langle E_\ell \rangle$, $\langle E^2_\ell \rangle$, $\langle E^3_\ell \rangle$; $\langle M^2_X \rangle$, $\langle M^4_X \rangle$, $\langle M^6_X \rangle$ ...

all as functions of the lower cut on charged lepton energy

Precision data on the photon spectrum in $B\to X_s+\gamma$ are important!
Theoretical status

Can aim at 1% level in $|V_{cb}|$ with a technical progress in theory:

need $\alpha_s$-corrections to power-suppressed Wilson coefficients

$|V_{ub}|$ ? – underway, 5% accuracy is realistic

A question: Should we trust theory that much?

- $\langle M_X^2 \rangle$ vs. $E_{\text{cut}}^\ell$

Robust OPE approach à la Wilson, $\mu = 1\text{GeV}$:

Data and expectations as of July 2003

Bigi, N.U. hep-ph/0308165
Gambino, N.U. hep-ph/0401063

BaBar
CLEO
DELPHI theory
Second mass$^2$ moment $\langle [M_X^2 - \langle M_X^2 \rangle]^2 \rangle$:

The most recent fit to all data: hep-ph/0507253

Good agreement where the right theory is used right

OPE works well even where can be expected to break down
What is still missing:

- $\alpha_s$-corrections to the power-suppressed Wilson coefficients: the principal limiting factor

- Is charm sufficiently heavy? we do not expand in $1/m_c$, yet

Effects of the nonperturbative four-quark expectation values with charm $\langle B|\bar{b}c\bar{c}b|B\rangle$ superficially resemble ‘Intrinsic Charm’

Required in the consistent OPE

see Benson et al., hep-ph/0302262

Analysis (Bigi, N.U., Zwicky, to appear):

In the $1/m_c$ expansion the effect appears at the sub-% level in $\Gamma_{sl}$, is expected below 0.5% due to cancellations

Experiment directly constrains the effect at 1 to 2% level

Expect improvement down to 0.5% where it would not affect precision of $V_{cb}$
A comprehensive fit including all moment measurements:
(by the professionals)

Experimental fit to all these data:

\[
V_{cb} = (4.190 \pm 0.044 \pm 0.053) \cdot 10^{-3}
\]

\[
4.179 \pm 0.048
\]

\( b \rightarrow c \ell \nu \) only
- SL decays yielded accurate $m_b$ itself \hspace{1cm} not obvious a priori

$$m_b = (4.591 \pm 0.038) \text{ GeV}$$
$$4.644 \pm 0.072$$ \hspace{1cm} $b \to c \ell \nu$ only

The combination $m_b - 0.74 m_c$ is determined with only a 15 MeV error bar!

$V_{cb}$ depends on nearly the same combination of $m_b$ and $m_c$ – that is why it is so accurate

Running ‘kinetic’ mass is an observable and has no intrinsic limitation on precision

**Theoretical expectation:** \hspace{1cm} $m_b(1 \text{ GeV}) = (4.57 \pm 0.06) \text{ GeV}$

Voloshin \hfill 1995–1996
Melnikov, Yelkhovsky \hfill Hoang \hfill 1998–1999
Beneke, Signer

$e^+e^- \to \gamma(1S, 2S, 3S, 4S, 5S)$
\hspace{1cm} moments of $\sigma(e^+e^- \to b\bar{b})$
\[ \mu_G^2 = \frac{1}{2M_B} \langle B | \bar{b} \left( \frac{i}{2} g_s \sigma_{\mu \nu} G^{\mu \nu} b \right) | B \rangle \quad \longleftrightarrow \quad \langle B | -g_s \vec{\sigma} b \vec{B}_{\text{chr}}(0) | B \rangle_{\text{QM}} \]

\[ \mu_\pi^2 = \frac{1}{2M_B} \langle B | \bar{b} (i\vec{D})^2 b | B \rangle \quad \longleftrightarrow \quad \langle B | \vec{p}_b^2 | B \rangle_{\text{QM}} \]

\[ \vec{p}_b \rightarrow \vec{\pi}_b = -i\vec{D} = -i\vec{\sigma} - g_s \vec{A} \]

\[ \mu_G^2 \text{ determines hyperfine splitting: } \quad M_{B^*} - M_B \simeq \frac{2 \mu_G^2}{3 m_b} \]

\[ \mu_G^2(1 \text{ GeV}) = 0.35^{+0.03}_{-0.02} \text{ GeV}^2 \quad \text{N.U. PLB 2002} \]

\[ \mu_\pi^2(\mu) > \mu_G^2(\mu) \quad \text{at any } \mu \quad \text{rigorous inequality} \quad \text{BSU; Voloshin 1993–1994} \]

**Theory (conservatively):** \[ \mu_\pi^2 \simeq (0.45 \pm 0.1) \text{ GeV}^2 \]

**The most recent experimental value**

\[ \mu_\pi^2 = 0.405 \pm 0.04 \text{ GeV}^2 \quad \text{hep-ph/0507253} \]

**Darwin expectation value also emerged of the right scale,**

\[ \rho_D^3 \simeq \frac{2}{3} \frac{(\mu_\pi^2)^2}{M_B - m_b} \simeq 0.2 \text{ GeV}^3 \quad \text{BSU; Pirjol & N.U. 1997} \]

**Fit value:** \[ \rho_D^3 = (0.17 \pm 0.025) \text{ GeV}^3 \]
Have an accurate and reliable determination of many HQ parameters from experiment

Extracting $|V_{cb}|$ from $\Gamma_{sl}(B)$ has good accuracy and solid grounds

Have precision checks of the OPE at the nonperturbative level

Overall there are many remarkable agreements with predictions

I think the most impressive is good consistency between $\langle M_X^2 \rangle$ and $\langle E_\ell \rangle$: A sensitive check of the nonperturbative sum rule for $M_B - m_b$

Important: the HQ values emerge in accord with the theoretical expectations: $m_b, \mu_\pi^2 > \mu_G^2, ...$ the right scale for $\rho_D^3$

Theory seems to work too well?

‘Theoretical correlations’

Need to check in a different environment:

consider $b \rightarrow$ light $q$ decays
Using $b\to c\ell\nu$ information for $b\to u\ell\nu$?

Contrary to naive nonrelativistic quark models, the heavy quark distribution function in QCD entering $b\to c\ell\nu$ is different from the one describing $b\to u\ell\nu$ or $b\to s+\gamma$ inclusive decays.

Nevertheless, the OPE relates the most important lowest moments, in particular those associated with $m_b$ and $\mu_\pi^2$. This may look not obvious when perturbative corrections are superimposed onto ‘primordial’ Fermi motion, but remains true.

The achieved precision in $b\to c$ is higher, and it can be utilized.

Predictions for $b\to u$ constrained rates benefit from the values of $m_b$, $\mu_\pi^2$, ... Eliminating them in favor of the light-cone distribution function alone counteracts skepticism about the OPE relations.

The same misconception underlies suggestions like those that a different ‘light-cone based’ scheme is better for $b\to q$ transitions.
• $b \to s + \gamma$ moments

There were problems when the OPE relations were blindly used with a high cut on $E_\gamma$

$$\langle E_\gamma \rangle = \frac{m_b}{2} + \ldots$$

$$\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = \frac{\mu_\pi^2}{12} + \ldots$$

A way to measure directly the HQ parameters?

Bottle neck: ‘Hardness’ $Q$ often gets too low with the cuts even in $b \to c \ell \nu$ $Q \simeq m_b - m_c$ for total widths, but $Q$ is below 1 GeV for $E_\ell > 1.7$ GeV

A complementary consideration suggests the expansion for $M_X^2$ loses sense for $E_{cut} \geq 1.7$ GeV

Terms appear $\propto e^{-\frac{Q}{\mu_{hadr}}}$

In $b \to s + \gamma$ $Q \simeq M_B - 2E_{min} \simeq 1.2$ GeV

if the cut is at $E_\gamma = 2$ GeV

Accounting for these biases yielded a good agreement between all measurements
Perturbative corrections with the explicit Wilsonian cutoff have been calculated including all orders in BLM

Benson, Bigi, N.U. hep-ph/0410080

**Good news:** complete $\alpha_s^2$ spectrum is available!

Melnikov, Mitov hep-ph/0505097

As expected, unknown perturbative corrections are small in the Wilsonian approach, e.g. $\delta m_b(1\text{ GeV}) \lesssim 5\text{ MeV}$

**BELLE 2004:** With $E_\gamma > 1.8\text{ GeV}$ cut *biases* are moderate

\[
\begin{align*}
\langle E_\gamma \rangle &= 2.292 \pm 0.026_{\text{stat}} \pm 0.034_{\text{sys}} \text{ GeV} \\
\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle &= 0.0305 \pm 0.0073_{\text{stat}} \pm 0.0063_{\text{sys}} \text{ GeV}^2
\end{align*}
\]

With the HQ values extracted from SL decays we would get

\[
\langle E_\gamma \rangle = 2.315 \text{ GeV} \quad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0325 \text{ GeV}^2
\]

**CLEO 2001:** $E_{\text{cut}} = 2\text{ GeV}$

\[
\begin{align*}
\langle E_\gamma \rangle &= 2.346 \pm 0.032_{\text{stat}} \pm 0.011_{\text{sys}} \text{ GeV} \\
\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle &= 0.0226 \pm 0.0066_{\text{stat}} \pm 0.0020_{\text{sys}} \text{ GeV}^2
\end{align*}
\]

vs.

\[
\begin{align*}
\langle E_\gamma \rangle = 2.345 \text{ GeV} \quad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.022 \text{ GeV}^2
\end{align*}
\]

**Quite consistent!**

**BaBar 2005:** $E_{\text{cut}} = 1.9\text{ GeV}$

\[
\begin{align*}
\langle E_\gamma \rangle &= 2.343 \pm 0.053_{\text{stat}} \pm 0.053_{\text{sys}} \text{ GeV} \\
\langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle &= 0.0325 \pm 0.016_{\text{stat}} \pm 0.011_{\text{sys}} \text{ GeV}^2
\end{align*}
\]

vs.

\[
\begin{align*}
\langle E_\gamma \rangle = 2.327 \text{ GeV} \quad \langle [E_\gamma - \langle E_\gamma \rangle]^2 \rangle = 0.0275 \text{ GeV}^2
\end{align*}
\]
Theory predictions (Wilsonian OPE plus bias corrections) based on HQP from $b \to c \ell \nu$ data vs. experiment
Besides $V_{cb}$ and $V_{ub}$, how do we benefit from knowing the heavy quark parameters?

Precise values of $m_b$, $m_c$ — for textures (future)

Today’s use: $B$ nonperturbative dynamics

OPE operating in terms of the universal QCD operators has comprehensive applications not limited to only inclusive decay rates

For instance, heavy quark sum rules — particularly constraining with the HQP values coming from experiment

**Good example:** bound $\varrho^2 > \frac{3}{4}$ N.U. 2000

If $\mu_{\pi}^2$ is close to $\mu_G^2$ there is also a strong upper bound for the IW slope! N.U. 2001

Assuming the spin sum rule is saturated at $\mu = 1$ GeV we have

$$\mu_{\pi}^2 - \mu_G^2 = 3 \tilde{\varepsilon}^2 \cdot (\varrho^2 - \frac{3}{4})$$

Quite a constraint: $$\left(\varrho^2 - \frac{3}{4}\right) = \frac{\mu_{\pi}^2 - \mu_G^2}{3\tilde{\varepsilon}^2} \lesssim 0.2 \ (0.4)$$

at $\mu_{\pi}^2 = 0.42 \ (0.5)$ GeV$^2$ since $\tilde{\varepsilon} > 0.35$ GeV

$\varrho^2$ is crucial in extrapolating the $B \rightarrow D(\ast)$ amplitudes to zero recoil.
Eventually this prediction has been confirmed in experiment:

\[ \rho^2 = 0.95 \pm 0.09 \]

One of the miracles of the proximity to the ‘BPS’ regime

\[ \mu_\pi^2 \approx \mu_G^2 \] is a special point for \( B \) and \( D \) mesons!

In the strict limit \( \rho^2 = \frac{3}{4} \)

Ultrarelativistic light cloud – antipode to NR quark models

Another application – \( B \to D \ell \nu \): expanding in \( \mu_\pi^2 - \mu_G^2 \) and using the analogue of the Ademollo-Gatto theorem which holds for the BPS expansion

\[
\frac{M_B + M_D}{2\sqrt{M_B M_D}} f_+(0) = 1.04 \pm 0.01 \pm 0.01
\]

All orders in \( 1/m \) in ‘BPS’, to \( 1/m^2 \cdot 1/\text{BPS}^2 \), \( \alpha_s^1 \)
A “$\frac{1}{2} > \frac{3}{2}$” puzzle

Heavy Quark Sum Rules $+$ the known size of $\mu_G^2$, now also of $\mu_\pi^2$ give much information

Spin sum rules strongly suggest that $\frac{3}{2} P$-wave states must dominate over $\frac{1}{2}$ states. This automatically happens in all quark models respecting QCD and Lorentz covariance

Orsay quark models
quark models on light front:
Cheng, Chua, Hwang, hep-ph/0310359

Theory:

The most natural solution of all HQSRs:

$\frac{3}{2}$ states at $\epsilon_{\frac{3}{2}} \approx 450$ MeV and $\tau_{\frac{3}{2}}^2 \approx 0.3$ while

$\tau_{\frac{1}{2}}^2 \approx 0.07 \div 0.12$ with $\epsilon_{\frac{1}{2}} \approx 300 \div 500$ MeV

Why?

Average $P$-wave excitation mass gap:

$$\bar{\epsilon}_P \approx \frac{2\mu_\pi^2}{3\Lambda} \approx 0.45 \text{ GeV}$$

$$\sqrt{\frac{\mu_\pi^2}{3(\rho^2 - \frac{1}{4})}} \approx 0.45 \text{ GeV}$$

Typical $\tau^2$:

$$\bar{\tau}^2 \approx \frac{1}{3} (\rho^2 - \frac{1}{4}) \approx 0.25$$

$$\frac{\Lambda}{6\epsilon_P} \approx 0.25$$

and $\tau_{1/2}^2 \ll \tau_{3/2}^2$ from the spin sum rules
Experiment: $\frac{3}{2}$ charm $P$-wave states are narrow and well identified, \{\(D_1\), \(D_2^*\)\}

According to the analysis by Ligeti et al. of the semileptonic decays, these seemed to contribute too little, $|\tau_{3/2}|^2 \approx 0.15$

There are some weak points in the data analysis

Wide structures routinely associated with the $\frac{1}{2}$-states \{\(D_0^*, D_1'\)\} may be more abundant and might even saturate the spin-singlet sum rules, but in aggregate the $\frac{1}{2}$-states have to be subdominant to the $\frac{3}{2}$-states!

The semileptonic data on charm excitations yield are still somewhat fluid

Nonleptonic decays $B \to D^{**}\pi$ are better but assume factorization

While possibly consistent for $\tau_{3/2}$, they used to give definitely too large $\tau_{1/2} \sim 0.4$ (even at $q^2 = 0$), based on $B^- \to D^{**}\pi^-$

This would contradict spin sum rules

Belle, hep-ex/0412072: $B^0 \to D^{**}\pi^-$ modes witness smallness of $\tau_{1/2}$. The data are consistent with the decays only into $\frac{3}{2}$-states, at the right rate!

The decays of $B^0$ are free from another contribution unrelated to $\tau$'s (the color-suppressed diagrams)

Theory predictions from the HQ sum rules seem to be confirmed
Heavy Quark Expansion proved to be a powerful tool and works great when applied properly.

It allows precision treatment of strong interaction effects required to unfold the underlying fundamental parameters from experimental measurements.

Comprehensive Heavy Quark Expansion uncovers nontrivial connection between the inclusive decays and the properties of individual heavy flavor hadrons. A number of them have been recently confirmed by high-quality data, including those from $B$-factories.

Heavy Quark Theory is alive and evolves to improve our understanding of QCD in heavy flavor hadrons.