Synchronization of Double Pendulums

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I. Introduction

Chaotic behavior is well-known in the motion of the double pendulum. The tiny difference between initial conditions will lead to great difference in the result and we can't anticipate it. This feature might vanish under some constraint. For example, we make two chaotic systems coupled. In this poster, we will present the synchronization(sync) of two chaotic systems(double pendulum).

II. Research Method

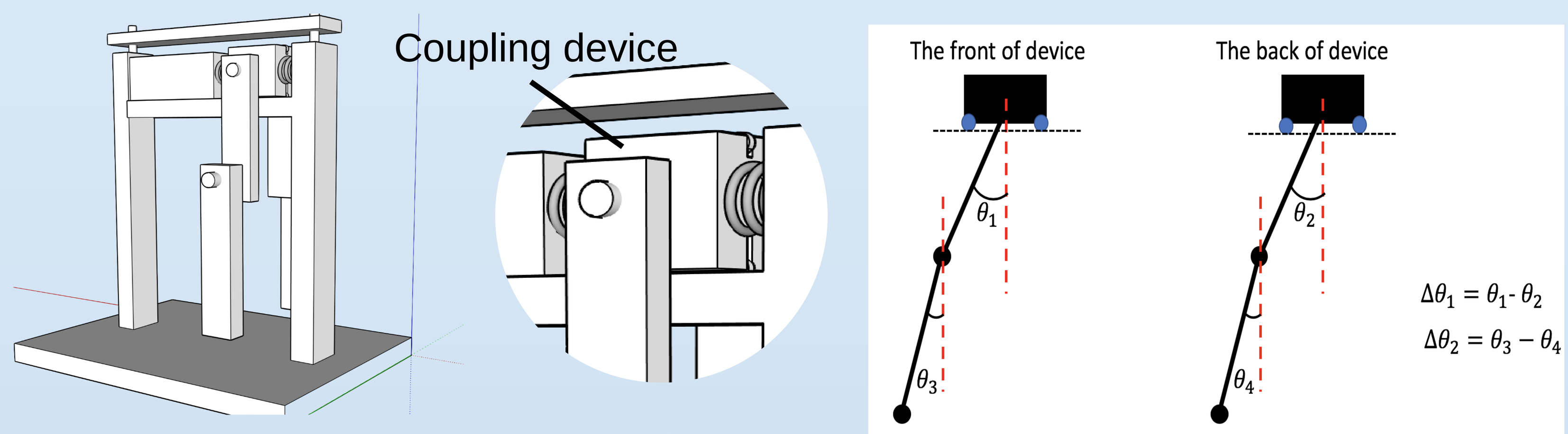

Fig. 1 Schematic diagram of apparatus
(a)The apparatus contains two double pendulums, which are connected to the cube (b) on the platform.
(c)The parameter of the sketch

In the experiment-al apparatus, we use a movable device to connect two double pendulums, in order to transfer energy between them. To analyze the phenomenon, the camera in both sides records the motion simultaneously. We acquire the track of two systems by Tracker and analyze the difference between two systems to recognize the sync.

III. Theoretical Model

Lyapunov exponent(λ) is a significant indicator of sensitivity.

$$d(t) = d_0 e^{\lambda t}$$

Where d(t) is the difference between two system as time.

The expression of the deviation function d(t) demonstrates the butterfly effect of the chaotic system. The difference must decrease as the occurrence of sync.

According to Lagrange's equation with dissipation, the dynamics of coupled pendulums is established.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} + \frac{\partial V}{\partial q_j} = Q_j$$

Lagrange's equation accurately describes the behavior of our experiment.

Reference:

[1] Kubachi Mateusz, *Dynamics of Coupled Pendulums*, Technical University of Lodz International Faculty of Engineering 2011.
[2] Chen Joe, Chaos from Simplicity: An Introduction to the Double Pendulum, University Canterbury Department of Mathematics and Statistics ,2008.

IV. Simulation

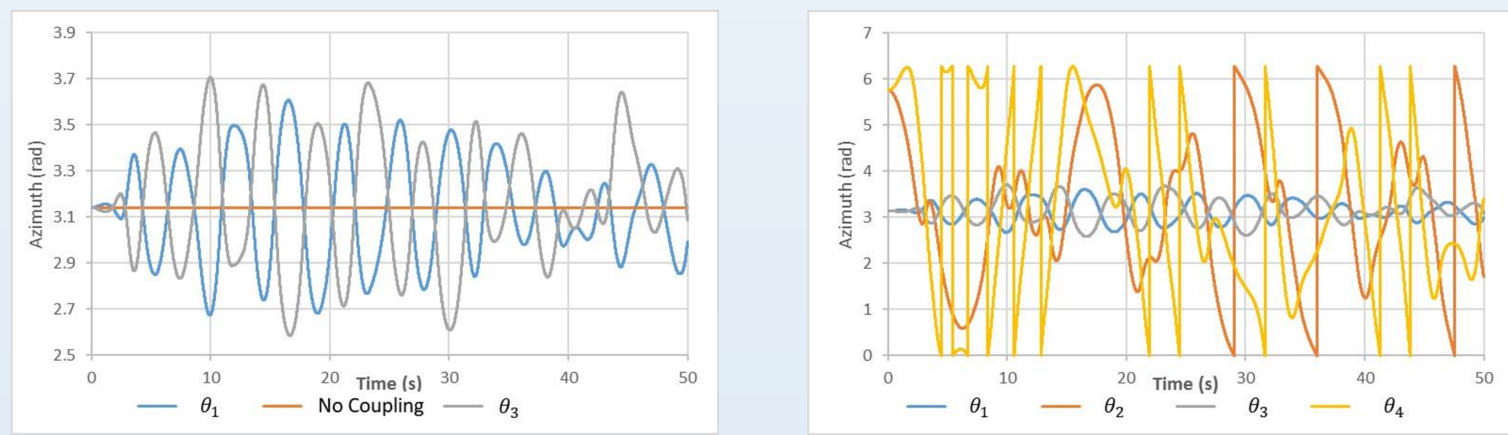

Fig. 2 Track of pendulums in simulation with initial conditions
System 1: $\theta_1 = 0°, \theta_3 = 0°$ System 2: $\theta_2 = 150°, \theta_4 = 150°$
(a) System 1 receives the energy through coupling device and grows in amplitude.
(b) Propagation of energy between two systems.

We are able to verify the validity of the coupling device in our experiment. Weak coupling influences the whole system exactly. The difference between systems decreases with the coupling(Fig.3).

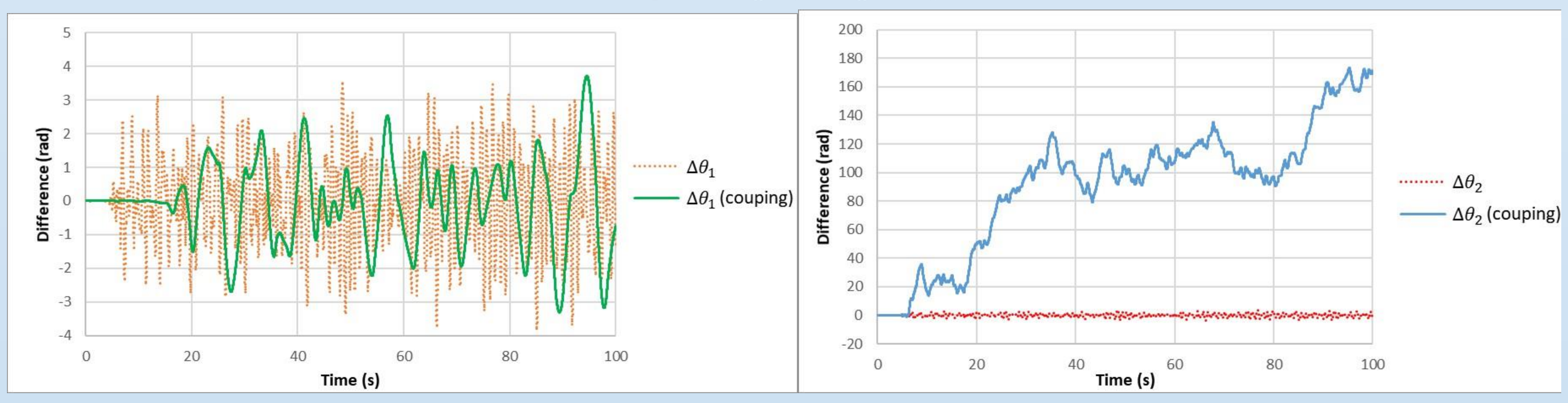

Fig. 3 Comparison of deviation between coupling and no coupling chaotic systems(double pendulums).

With coupling, the deviation of two systems does not increase as a general chaotic system. The difference remains constant under the coupling constraint.

V. Result and Discussion

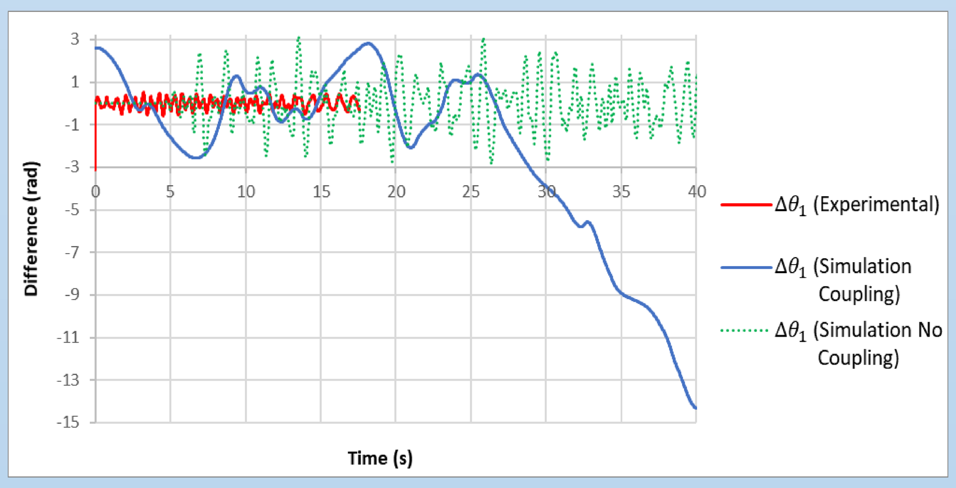

Fig. 4.a

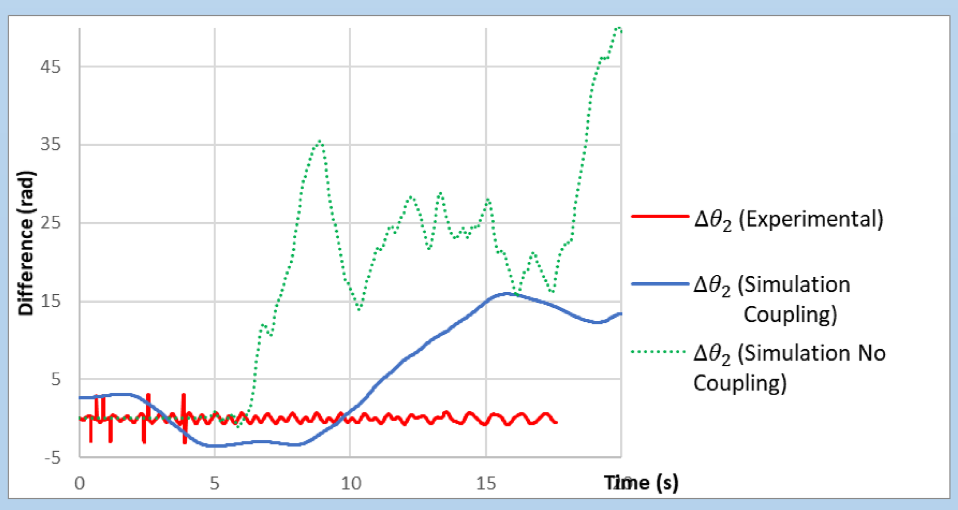

Fig. 4.b
Fig. 4 Difference comparison
(a) Upper pendulum
(b) Lower pendulum

The chaotic behavior leads to the growth in deviation. We found that the experimental deviation has a tendency similar to simulation with coupling. The ascent of deviation does not happen. However, the damping in the experi-ment might also cause the elimination of deviation so we analyze the deviation with increasing damping by simulation. The magnitude of damping does nothing to the consequence.

V. Conclusion

1. The coupling effectively suppresses the growth of deviation
2. The damping doesn't influence the experiment.

Appendix:

Experimental parameters

	System 1		System 2		Pulley
	Upper	Lower	Upper	Lower	
Mass(g)	84.37	83.28	84.56	83.16	100.24
Length(cm)	20.3	20.5	20.2	20.5	