Momentum transfer in viscoelastic medium

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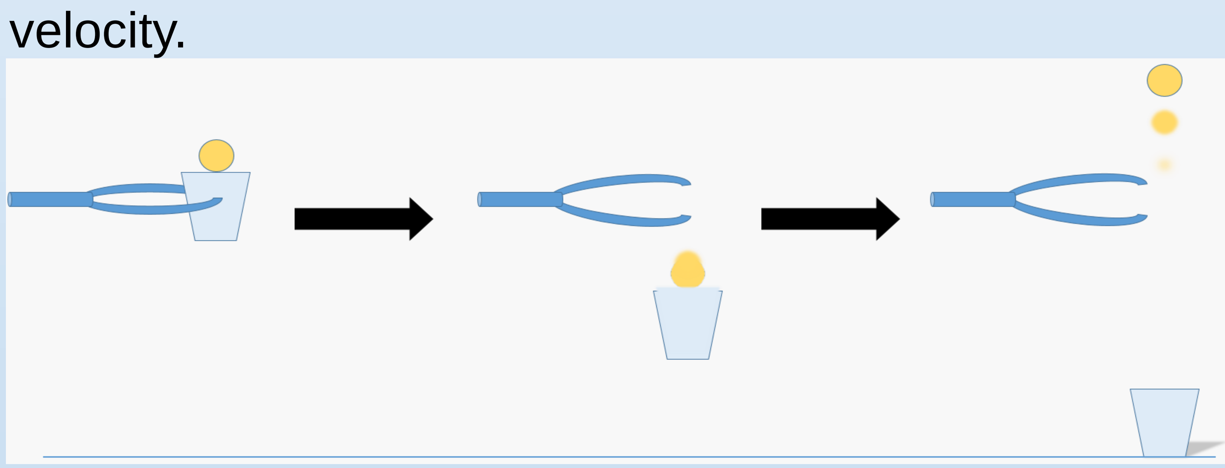
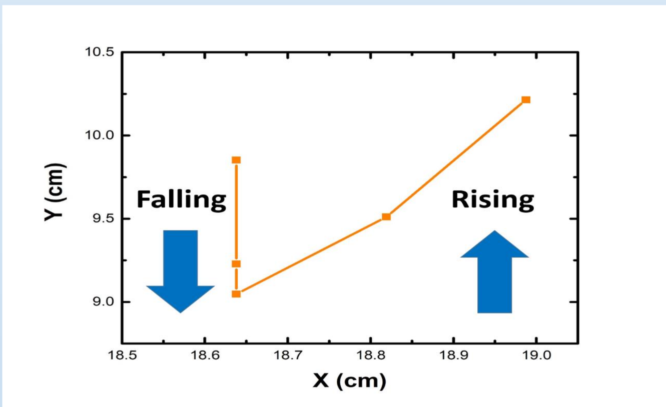
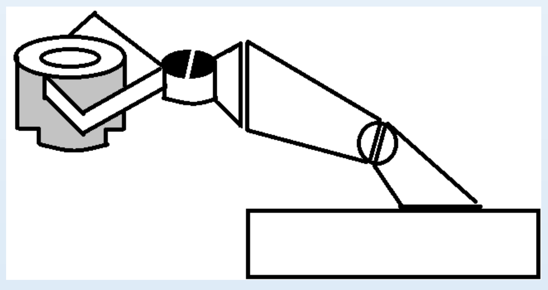
Introduction

Throwing a ball to the ground, it will rebound to the same height. However, put the ball onto a cup filled with water and throw it, the cup will stop at ground and the ball will rebound much higher. It is based on the conservation of momentum. We are curious about how will the ball react if we use different liquid. We used the jelly with different density, and changing temperature to adjust viscoelasticity. Viscoelasticity is the combination of viscosity and elasticity. We measure the falling and rising velocity, to observe how viscoelasticity affect the ball. We also observe Brownian motion of water and jelly, and calculate mean square distance(MSD). With MSD, we can get the diffusion coefficient and viscosity, and make a connect to the falling and rising velocity.





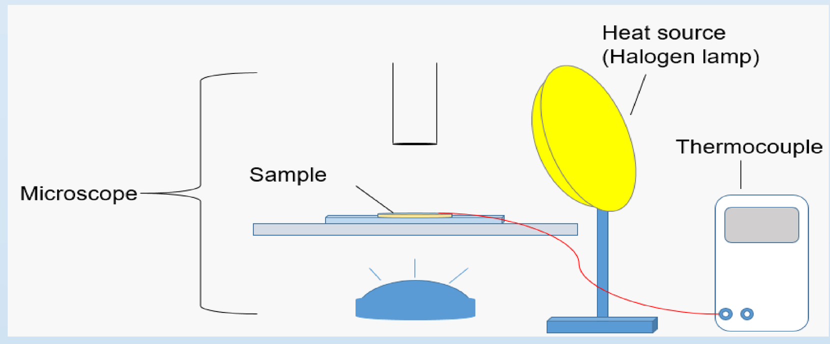
Setup and Methods



In order to observe the phenomenon under the macroscopic, the mechanical arm with Arduino program was set to catch the cup. The frame rate of camera is 480 fps. With position of data, we can transform the position with pixel on the computer and get the real velocity of the ball. For accuracy, we take average of both 2 pictures before the stable state.

For microscopic view, we use optical microscope to observe the Brownian motion in the water and viscoelastic fluid at different temperature.

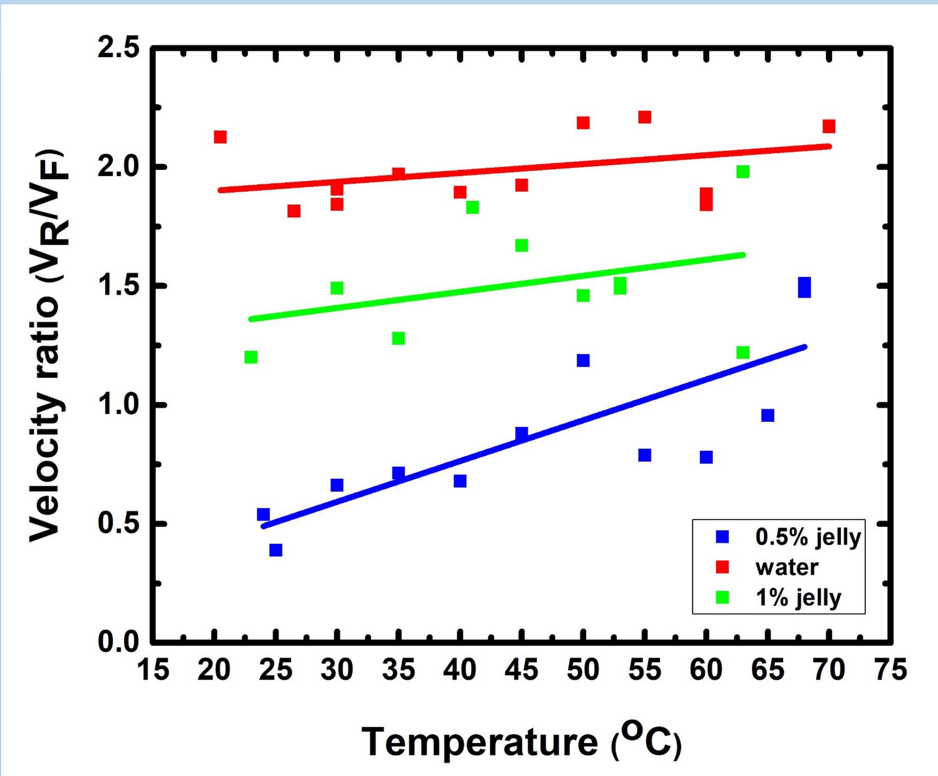
With constant heat source, we can control the temperature by changing the distance between the stage and heat source.



Result

Macroscopic phenomenon

Viscoelasticity includes viscosity and elasticity. Viscosity is resistance of deformation under shear stress, and can known as self friction. Elasticity is an ability of resist deformation and revert to original state while the force is remove.



- The ratio of jelly is smaller than water.
- The higher the temperature, the higher the ratio, but have lower viscosity.

Microscopic phenomenon

By the experiment phenomenon, we found that this liquid has both viscosity and elasticity depending on the time and temperature. To analysis the Brownian motion, we take MSD (mean-square-displacement) first in 2-D space and MSD follows：

$$MSD(\tau) =< \Delta r(\tau)^2 >=< [r(t+\tau) - r(t)^2] >$$

$$\text{where } \ r(\tau) = x(\tau)^2 + y(\tau)^2$$

Based on the principle of Boltzmann, the stress-strain relationship in frequency domain yield

$$\tilde{\sigma}(\omega) = E^*(\omega)\tilde{\varepsilon}(\omega)$$

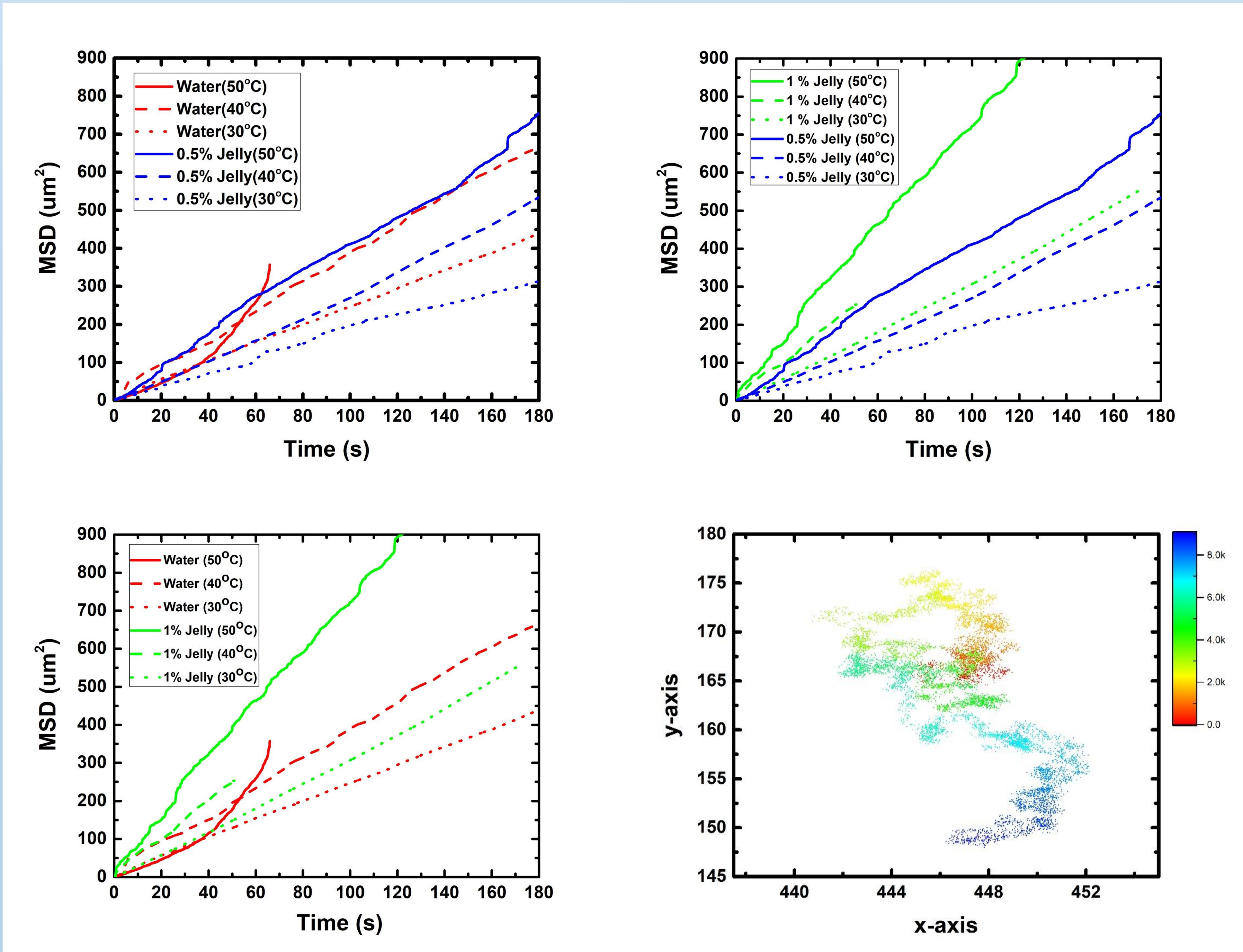
$$where\ \sigma\ is\ stress, E\ is\ relaxed\ modulus\ and\ \varepsilon\ is\ strain$$

and the loss factor $\eta(\omega)$ can be defined as

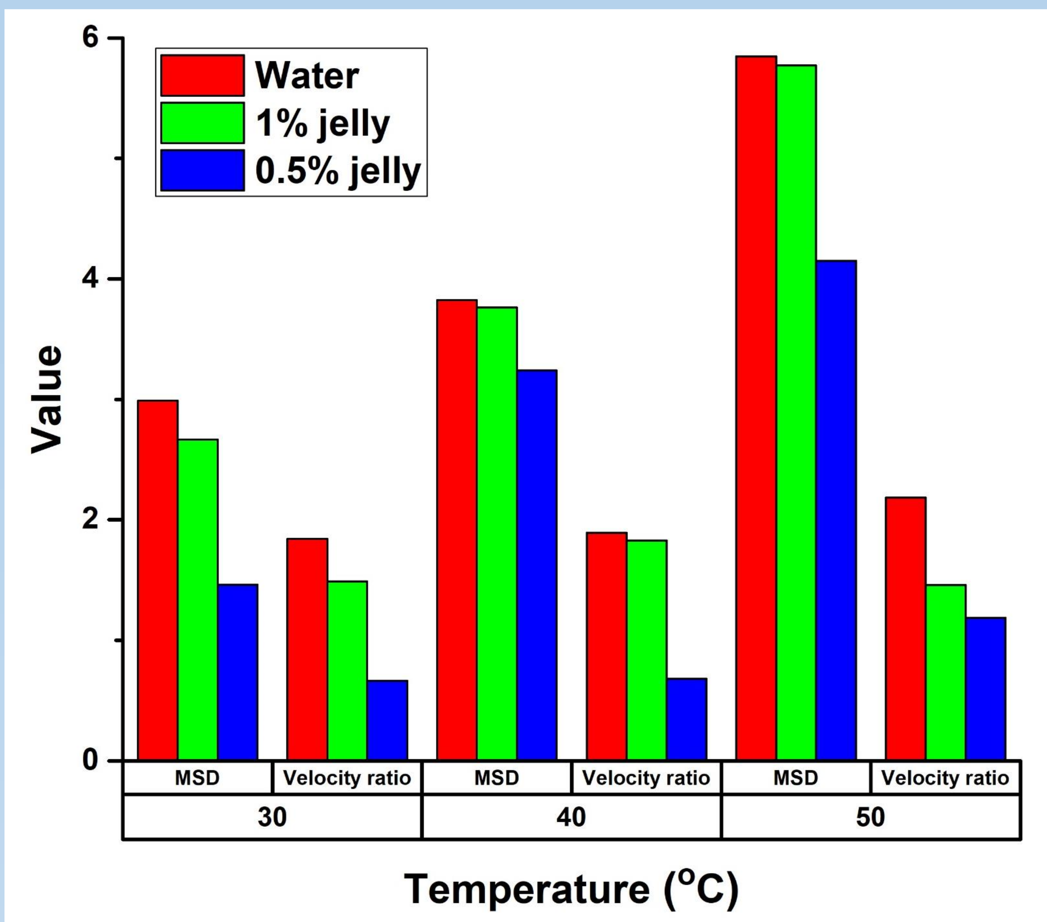
$$\eta(\omega) = \frac{E''(\omega)}{E'(\omega)},$$

$$\text{where } E''\ and\ E'are\ storage\ and\ loss\ modulus$$

In viscoelasticity we can take FFT on MSD to get the parameter of E'' and E' which are the representations of elastic and viscous properties in viscoelasticity in our experiment.



MSD can represent the displacement of particles and its slope means the viscosity. When the slope increases, the viscosity of the fluid will be smaller. Therefore, the viscosity between water, 0.5% jelly, and 1% jelly in 30, 40 and 50°C are studied. According to Boltzmann principle and the result of macroscopic experiment, we should find the elastic behavior of jelly. However, we only find the lower frequency about 30Hz of MSD by using FFT in frequency domain. It means that elastic properties are only shown in macroscopic experiment.



We put the slope of MSD and velocity ratio to combine macroscopic and microscopic conditions. It shows 0.5% jelly is more viscous than 1%, and 1% is more viscous than water, in both macroscopic and microscopic experiments.

Conclusion

1. The value of the viscosity from high to low are 0.5%, 1% and water.
2. In microscopic, increasing viscosity will decrease the MSD.
3. Time scale of our microscopic experiment is too large to see the elastic relaxation time. That's why we could not see the storage modulus after MSD or MSD Fourier transformation. Without the storage modulus, we could not get the loss factor.

Reference

1.Modelling viscoelastic materials whose storage modulus is constant with frequency, Fernando,C., María Jesús E., 2006, Department of Mechanical Engineering, Mondragon Unibertsitatea