The Climbing equation for the Leidenfrost droplet

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I. Introduction

Leidenfrost effect shows that droplets can move like there is no friction upon threshold temperature. This plate has to be heated by a hot plate about 250°C. Our experiment is about observing droplets movement on the aluminum sawtooth plane. Moreover, because of the sawtooth surface, the droplet is pushed forward by the reactive force of steam which is evaporate from droplets. Some people have done this experiment and have derived the equation of motion of droplets, but they only did the case of placing the aluminum plate horizontally. We decided to do the slant version and find out the equation.

II. Apparatus And Method

We use the hot plate to heat our sawtooth plate around 250°C, and Figure 1 is our Configuration.

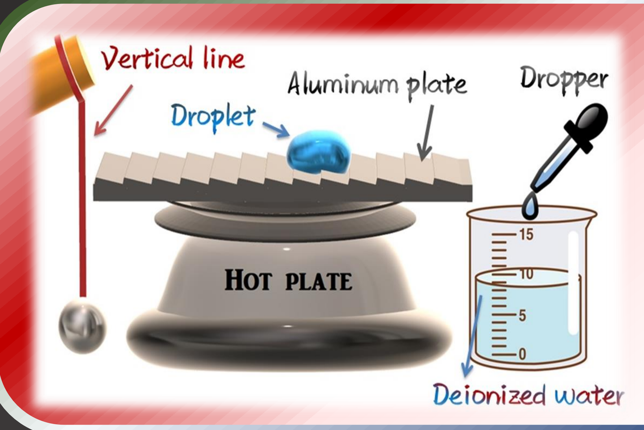

Fig. 1.
(a) Vertical line is used in tracker to measure the angle of the aluminum plate. (b) We use deionized water to avoid the precipitation of Ca and Mg ion on hot surface of the aluminum plate.

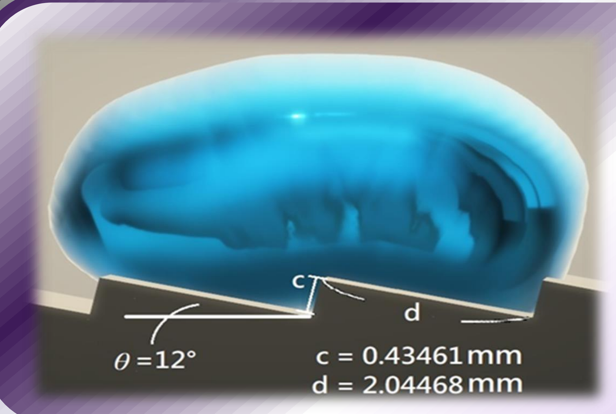

Fig. 2.
We referred to other people's paper to know that the water droplet has its maximum velocity at about 12 degrees of sawtooth angle. This is the actual size of our sawtooth plate.

The basic Newton equation is as below. The parameter **a** is given by the reactive force, **β** is retarding constant, ***φ*** is the angle of the slope, and we assume that **a** and **β** are independent of m：

$$F(t) = -\beta v_x(t) + ma - mgsin\varphi \qquad \text{[Eq. 1]}$$

$$ma = 0.5A_{eff}h\left|\frac{dP}{dx}\right|cos\theta\,, -\beta v_x = -(\eta A_{eff}/h)v_x \qquad \text{[Eq. 2]}$$

A_{eff} is the effective area of vapor reactive force; h is the thickness of the vapor; $\left|\frac{dP}{dx}\right|$ is pressure difference between the front and back end of droplets; θ is defined by the sawtooth angle (see Fig.2); η is the viscosity coefficient.

After solving it, we obtain the function of the velocity, and V_0 is initial velocity：

$$v_x(t) = \left[V_0 - \frac{a}{\beta/m} + \frac{gsin\varphi}{\beta/m}\right]e^{-\frac{\beta t}{m}} + \frac{a}{\beta/m} - \frac{gsin\varphi}{\beta/m} \qquad \text{[Eq. 3]}$$

And we find that **a** and **β** change with the ***φ***, so we modify our basic equation：(We set $V_0 = 0$) [Eq.3], [Eq.4]

$$\begin{cases} F(\varphi,t) = m\dfrac{dv_x(\varphi,t)}{dt} = -\beta(\varphi)v_x(\varphi,t) + ma(\varphi) - mgsin\varphi \\ v_x(\varphi,t) = m\left(-\dfrac{a(\varphi)}{\beta(\varphi)} + \dfrac{gsin\varphi}{\beta(\varphi)}\right)e^{\frac{-\beta(\varphi)t}{m}} + m\left(\dfrac{a(\varphi)}{\beta(\varphi)} - \dfrac{gsin\varphi}{\beta(\varphi)}\right) \end{cases}$$

III. Experimental Results

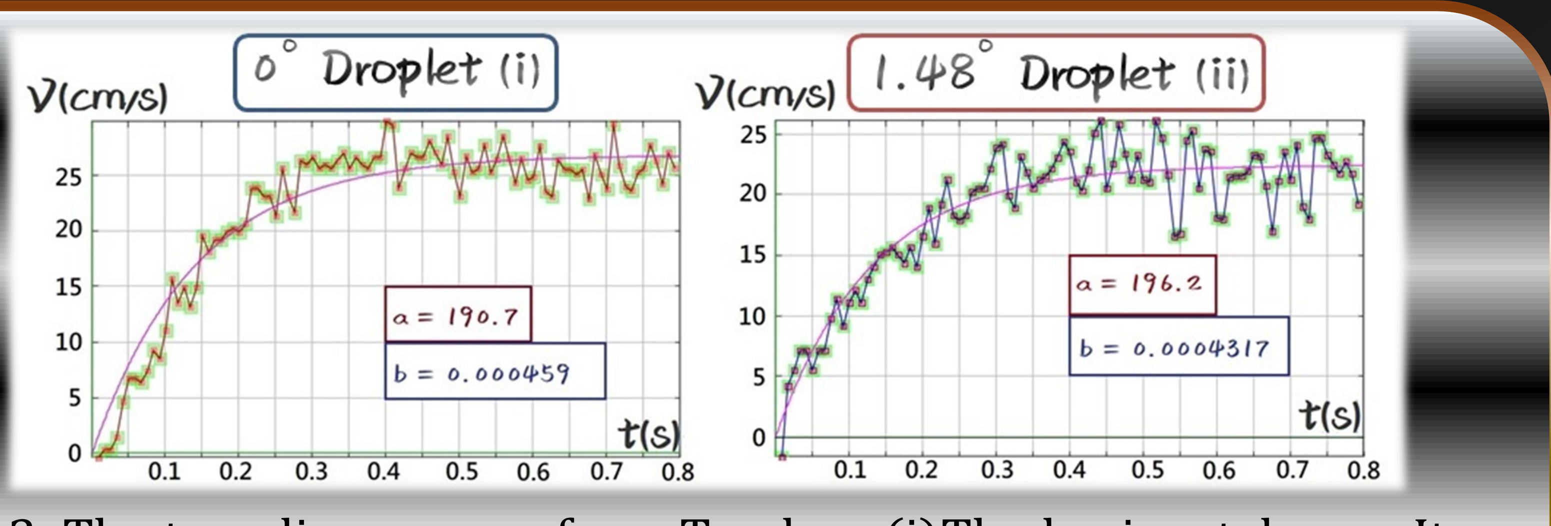

Fig. 3. The two diagram are from Tracker. (i)The horizontal case. It can be fitted well by the original equation. (ii) The tilted case. We fit it with Eq.4 perfectly.

We shoot 5 videos for different tilting angle and use the software, name Tracker to trace all the droplets. Figure 3 is the diagrams we track. The left one tells us that we can fit the velocity equation easily with the 0° case; the right one (1.48° case) tell us our equation (Eq.4) is correct.

With different droplets and angles, we can plot Fig. 4 (with Table 1.) . We have 6 droplets for 0° , 4 droplets for 1.48°, 5 droplets for both 2.61°, 3.67°, and 0.4°. Fig. 4 shows us that the values of **a** and **β** would decrease while **φ** increase.

Angle	average a	average beta
0	204.23	0.00061972
0.4	215.00	0.00055838
1.48	202.93	0.00038468
2.61	183.54	0.00040008
3.67	182.94	0.00041262
Average	194.36	0.00051271

Table 1. The mean value of **a**, **β** in different tilting angle.

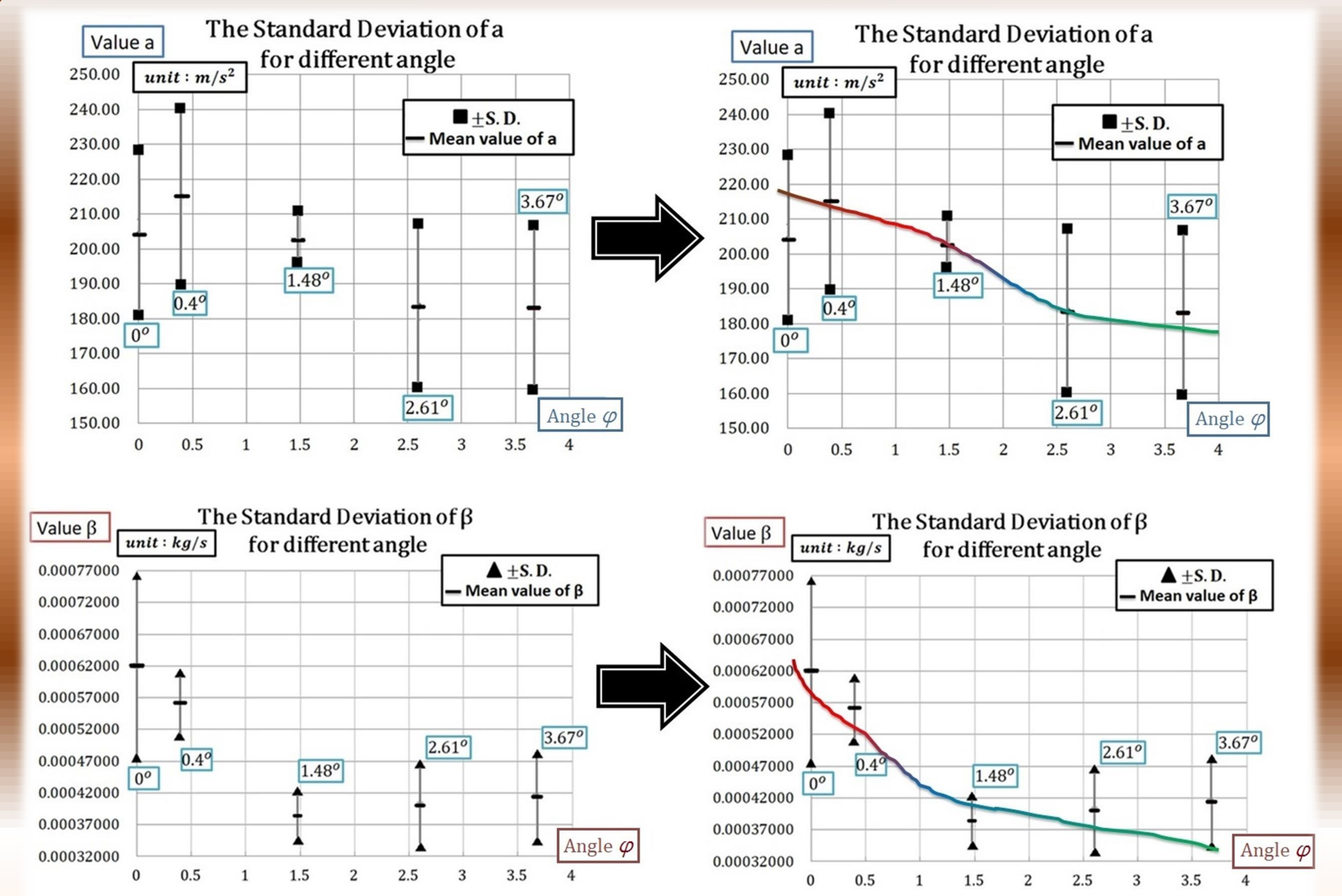

Fig. 4. The calculated values of **a** and **β**. We plot the approximate trendline for the two diagrams.

IV. Discussion

As shown in the Fig. 5, the contact area of water droplets in the horizontal case to the long side of the sawtooth is larger, and the effective contact area on A_{eff} is also larger. In contrast, in the tilted case, as the water droplets climb up, the space at the concave between the long side and the short side will become larger, making the contact area between the water droplets and the long side smaller. Therefore, we can obtained the conclusion that **a** and **β**(see eq.1 and eq.2) decrease as the angle of sawtooth plate increasing.

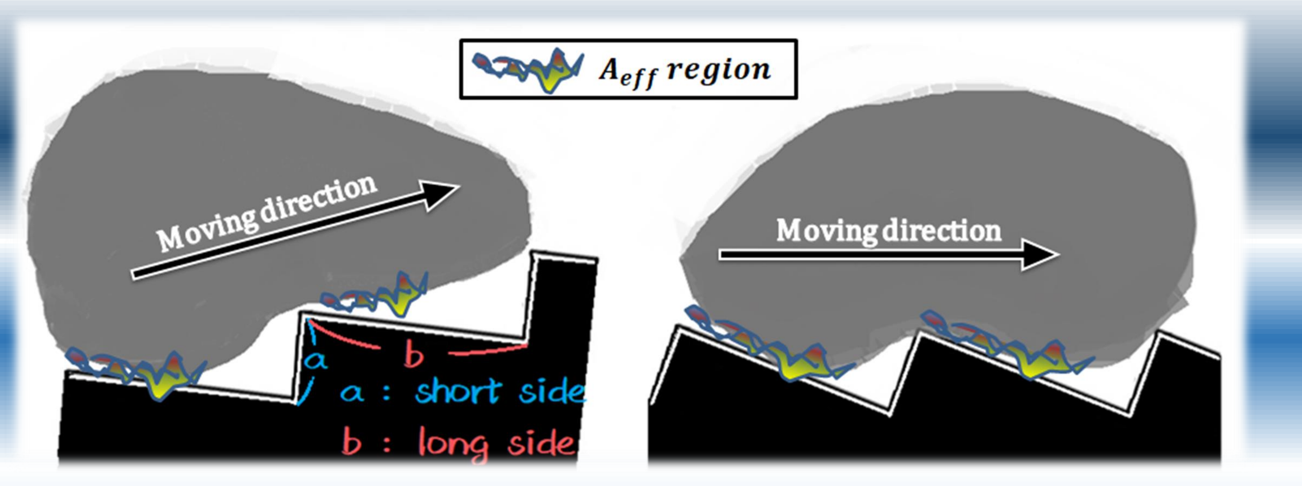

Fig. 5. The A_{eff} region of droplets in two different case.

V. Conclusion

Although we don't find a nice fitting function of **a(φ)** and **β(φ)**, we can still observe that **a**, **β** decline gradually while **φ** increase. The reason is that A_{eff} and $\left|\frac{dP}{dx}\right|$ of Eq.2 would change with **φ**. Base on the definition of Eq.2 and Eq.3, we predict that the behavior of the curve, **a(φ)** and **β(φ)**, would decline. Therefore, our predictions succeed to match with the experimental results of ours.

Reference:

[1] https://activity.ntsec.gov.tw/activity/race-2/2011/pdf/140003.pdf
[2] H. Linke, B.J. Alenman, L.D. Melling, Self-Propelled Leidenfrost Droplets PRL 96, 154502 (2006)