Determination of gravitational constant by pendulum-capacitor-based relaxation oscillators

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Abstract:

In this project, we purpose a novel way to measure the gravitational constant by using pendulum capacitor based relaxation oscillator (RO). The idea is referred to the work of Harold V. parks, et. al.[3]. The pendulum and the metal plate form a capacitor, and the frequency of the RO is determined by fixed resistor and the suspended capacitor. Gravity between the heavy object and the pendulum pulls the pendulum, as a result, changes the capacitance and the frequency of the RO as well. Moreover, thermal drift should be take into account. A same dimension but fixed pendulum capacitor and RO can be used to compensate thermal drift and common-mode noise by subtracting the frequency between two oscillators.

History:

In 1797 Cavendish used the force balance between the gravity of the object and the torque of the torsion wire to calculate the gravitational constant. He gets the torque by observing the rotational angle of the stick.

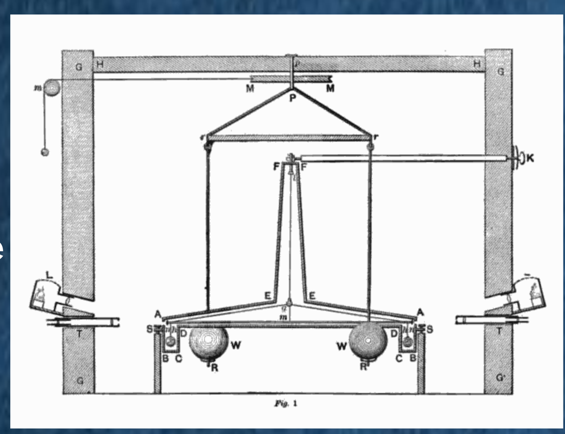
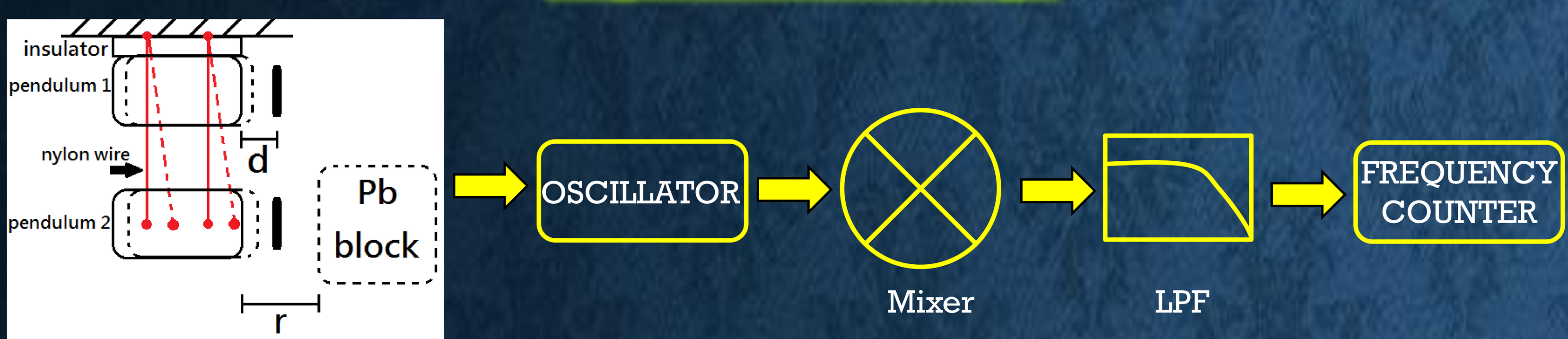


fig. 1 Cavendish experiment set-up

Experiment Set-up:



1. To cancel the effect of thermal expansion, we need two oscillators in the same condition. Thus, we set two brass cylinders and copper plates to be capacitors. The cylinder (a) was locked on the top, and the other one (b) was suspended by four nylon wires to control it only to shake forward and backward. The other side are two copper plates which were made on a printed circuit board (fig. 2).
2. We use 74HC14 (Hex Schmitt trigger) to build two relaxation oscillators.

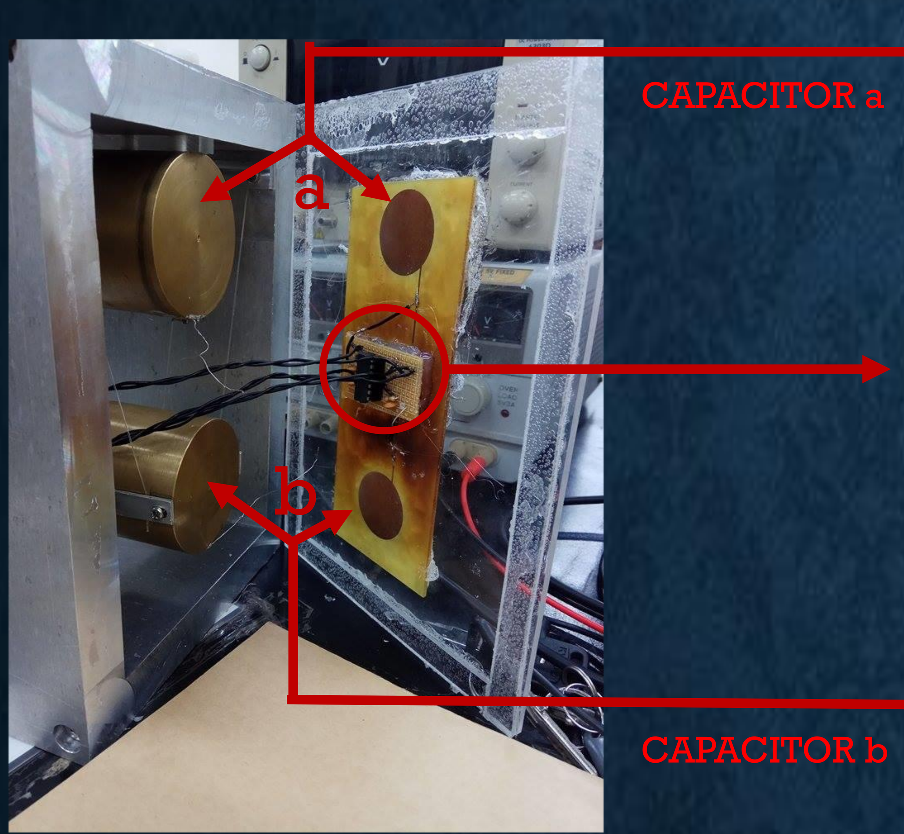


fig. 2 experiment set-up

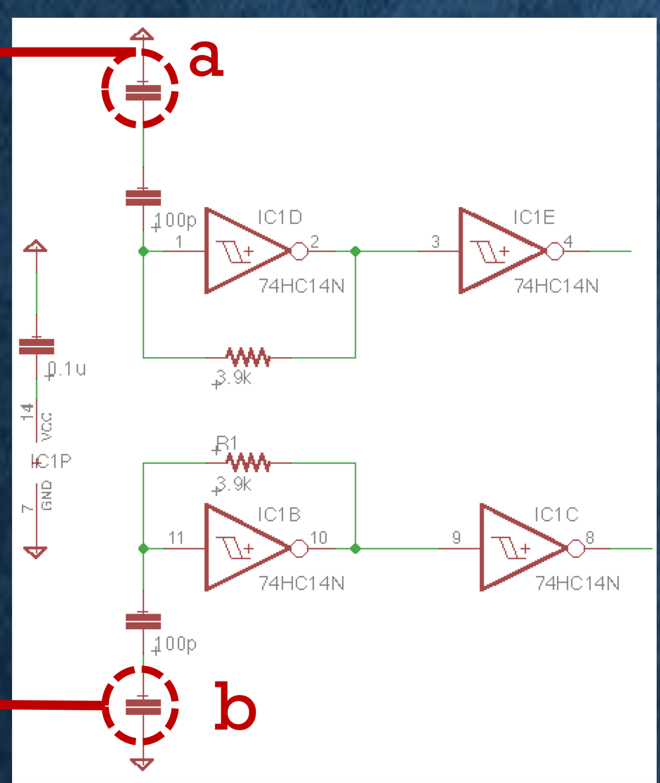


fig. 3 oscillator circuit

Size:	
Cylinder: $\phi 52.5\ mm,\ height\ 52.5\ mm$	Pb block: $100\times100\times100\ mm^3$
Area of capacitor: $225\pi\ mm^2$	Distance of two objects r= $11.5\ mm$
distance of capacitor $d=5.5\ mm$	Wire length $L=130\ mm$

3. Since we want to observe the change of the difference between two frequency, we use a double-balanced mixer(SA612) and two low-pass filters to get the difference frequency of two oscillators.

4. We use the frequency counter(Agilent 53230A) which is 12 significant digits to count the low frequency signal(f_1-f_2).

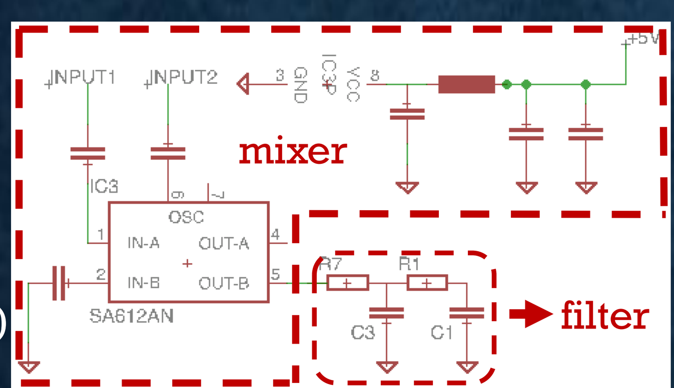


fig. 4 mixer and LPF circuit

Estimation:

In a 74HC14 RC oscillator, the relation between frequency(f) and capacitance(C) is $f=\frac{1}{0.8RC}$ [2], where R is the fixed resistance in the circuit and capacitance $C=\frac{\varepsilon A}{d}$ (ε: the air permittivity). By these equations, we can find out:

$$f_1-f_2=\Delta f=\frac{d_1-d_2}{0.8R\varepsilon A} \quad \text{(eq. 1).}$$

On the other side, by fig. 4, the relation between the force (F) and the weight of the brass cylinder (mg) is $F=\frac{mg\Delta d}{L}$. Substitute eq. 1, we can get:

$$F=\frac{0.8mg\Delta fR\varepsilon A}{L} \quad \text{(eq. 2).}$$

In general, the gravitational function is:

$$F=\frac{GMm}{r^2} \quad \text{(eq. 3).}$$

However, since we use brass cylinder and Pb block but not two mass points in our experiment, we need to do some corrections. We cut each two objects into 1000 cubes (the mass become dM and dm), and consider them as mass points.

After calculate the force of each cubes, we times $\cos\theta$ and $\cos\emptyset$ because we only consider the effect of the force in horizontal(x-axis). After these corrections, the gravitational force of between each cubes becomes:

$$F(x,y,z;a,b,c)=\frac{GdMdm}{\underbrace{(x-a)^2+(y-b)^2+(z-c)^2}_{r^2}}\underbrace{\frac{(x-a)}{\sqrt{(x-a)^2+(y-b)^2}}}_{\cos\theta}\underbrace{\frac{(x-a)}{\sqrt{(x-a)^2-(z-c)^2}}}_{\cos\emptyset}$$

(x, y, z) is the Pb block, (a, b, c) is the coordinate of the brass cylinder. a-axis and x-axis are in the coordinate of the same level.

parameters:

$R=3.9k\ \Omega$	$\varepsilon=8.55\times10^{-12}\ \frac{F}{m}$	$A=7.0686\times10^{-4}\ m^2$
$M=11.34\ kg$	$m=0.9717\ kg$	$G=6.67\times10^{-11}\ m^3kg^{-1}s^{-2}$

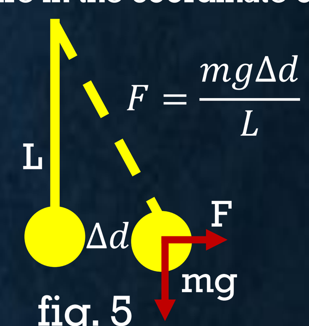


fig. 5

The result of estimation :
$F=8.6875\times10^{-8}\ N$ $\quad \Delta d=1.186\times10^{-9}\ m$ $\quad \Delta f=60.72\ Hz$

Method:

We use a Pb block($10\times10\times10\ cm^3$) as our heavy object to attract the pendulum. We let it approach capacitor (b) to change the oscillation frequency. Then we use counter(Agilent 53230A) to count the frequency before (f_b) and after (f_a) the Pb block leaves. We take f_b-f_a (fig. 6) to calculate the gravitational constant.

Result:

Fig. 6 shows a snapshot of one of our results, we take two mean values as the stable frequency before and after the Pb block leaves. After our calculation, we got the data below. (table. 1)

Δf (Hz)	Δd (m)	G ($m^3kg^{-1}s^{-2}$)
$7.69k$	1.88×10^{-7}	1.056×10^{-8}
$8.46k$	2.07×10^{-7}	1.162×10^{-8}
$7.78k$	1.90×10^{-7}	1.069×10^{-8}
$1.69k$	4.11×10^{-7}	2.316×10^{-8}
$7.07k$	1.73×10^{-7}	9.718×10^{-9}

table. 1 experimental results

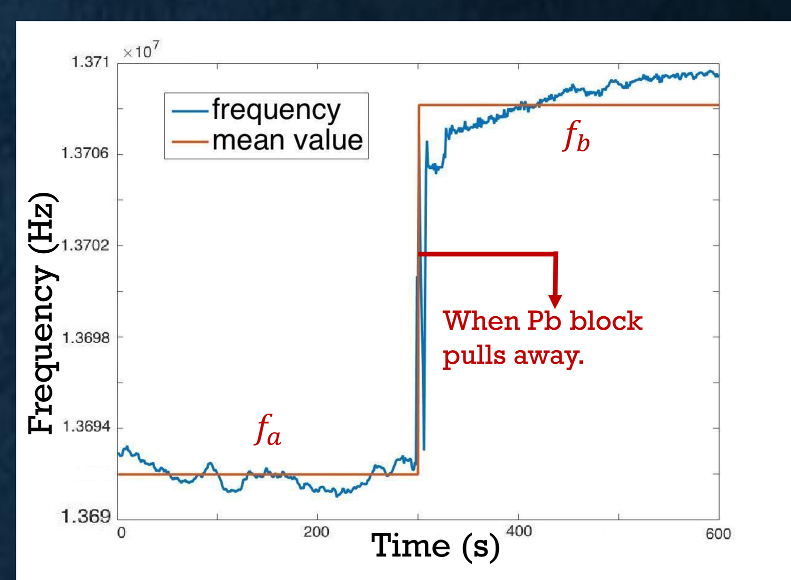


fig. 6

Discussion:

The results of our experiment have difference from our estimation. We speculate that we cannot measure the correct gravitational constant because of the following reasons:

1. While mixing f_1 and f_2, we observed that cross-talk between two oscillators made the mixer work abnormally. Thus, we did not mix the f_1 and f_2. That make us cannot cancel the effect of thermal expansion and other noise.
2. We did not prevent our set-up from all the tremor caused by wind, steps, voice and shaking of the building. Thus, it is hard for us to maintain the set-up stably.

Reference:

[1] https://en.wikipedia.org/wiki/Cavendish_experiment
[2] http://assets.nexperia.com/documents/data-sheet/74HC_HCT14.pdf
[3] Harold V. Parks, James E. Faller, "A simple pendulum laser interferometer for determining the gravitational constant", Philos Trans A Math Phys Eng Sci. 2014 Oct 13.

Acknowledgement:

Thanks Wang-Yau Cheng laboratory of comb laser based spectroscopy for lending us frequency counter.