Entropy Behaviors in Microscopic States With RC Circuit

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Purpose

- To study microscopic behaviors of entropy production through voltage fluctuation in RC circuits.
- Equilibrium state.
  - Observe the Johnson-Nyquist noise in the equilibrium state.
- Non-equilibrium steady state, NESS.
  - To prove the 2nd law of thermodynamics is not applicable to in microscopic.
  - To show the Fluctuation Theorem can be described by $ln(\frac{p(+X_\tau)}{p(-X_\tau)}) \rightarrow \frac{a}{k_B T}\ for\ \tau \rightarrow \infty$ (1) .
  - Setup a hand-made amplifier to get our data.

Johnson-Nyquist noise (RC circuits dynamics)

Johnson-Nyquist noise is an electronic noise generated by the thermal agitation of the electrons inside the resistance. Moreover, it is approximately white in an ideal resistor. The amplitude distribution will than be Gaussian. We can also see that the level of the noise by the power spectral density (PSD).

The dynamics of Brownian motion and RC circuit

Brownian motion described by Langevin equation is like

$$\alpha \frac{dx_t}{dt} - \xi_t - k(x_t - v^* t) = 0$$

x_t: position at time t.
α: friction coefficient.
k: strength of the harmonic potential induced by the optical tweezer.
v^*:constant speed which the tweezer is moved.
ξ_t: represents a Gaussian white noise.

We can analogize to RC circuit considering thermal fluctuation,

$$R\frac{dq_t}{dt} + \delta V_t - \frac{1}{C}(q_t - It) = 0$$

R, C: resister and the capacitor.
I: injected current.
dq/dt: current go through the resistor.
δV_t: Gaussian random noise.

TABLE 1. The analogy of the dynamic of a RC circuit and a Brownian particle.

Brownian particle	ξ_t	α	Xt	vt	k	v*
RC circuit	$-\delta V_t$	R	qt	it	1/C	I

Equilibrium state

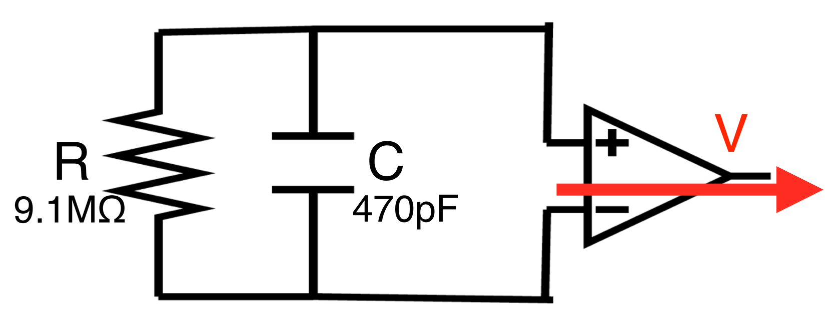

FIG.1 model circuit 1: We used the AD-DA card to measure the voltage change (rad line) between the resistor.

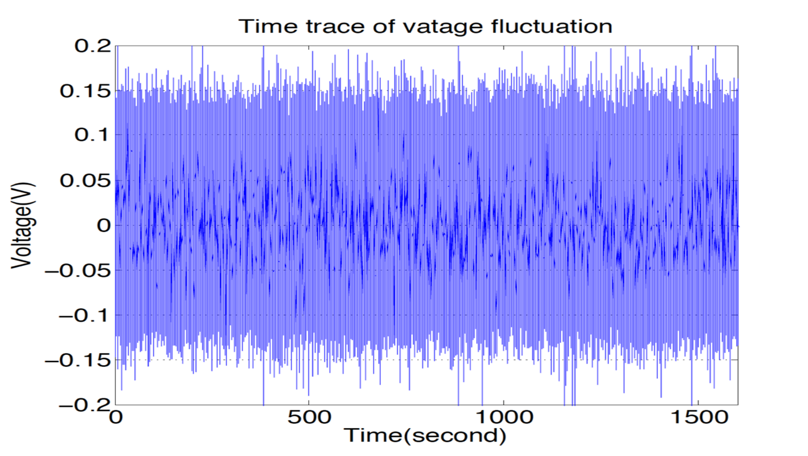

FIG.2 Time trace of voltage fluctuation: sampling frequency 8192Hz, sampling time 30min, gain 10000, number of sample 7.371×10^6.

Results

1. Probability density function (PDF) (FIG.3): By the Fluctuation-Dissipation Theorem, we know the the standard deviation of the voltage fluctuation equals to $\sqrt{k_B T/C}$.
2. Power spectrum density (psd) (FIG.4): By the fluctuation-dissipation theorem, we know the the standard deviation of the fluctuated V equals to $\sqrt{4k_B TR}$.

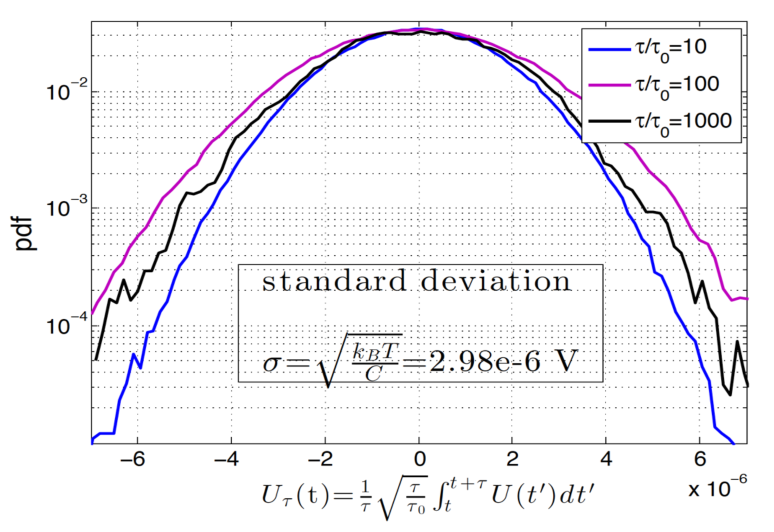

FIG.3 Probability density function shows the Gaussian distribution of the thermal noise, an the equivalent capacitance.

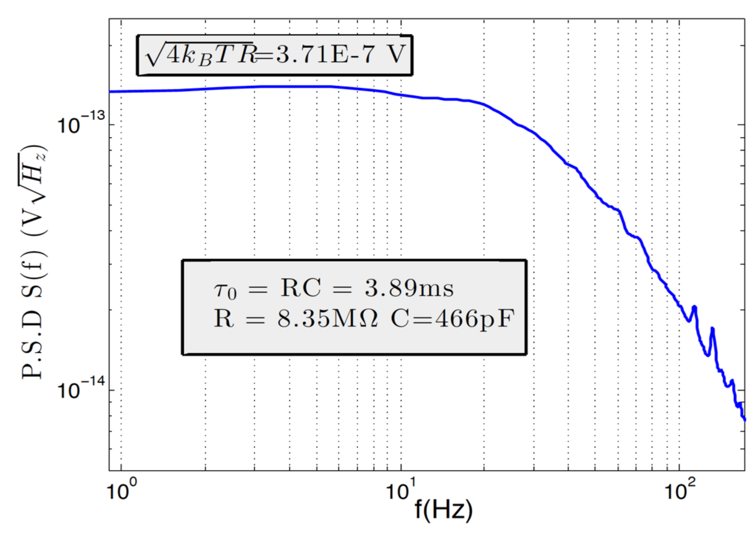

FIG.4 Power spectral density shows the intensity of the thermal noise and the equivalent resistance.

We call $\langle g \rangle_\tau(t) = (1/\tau)\int_t^{t+\tau} g(t')dt'$ the time-averaged value of a function g over time τ.

Handmade amplifier

To realize our measurement, we need to amplify our signal and isolate our system from environmental noise. Therefore, we place our model circuit into an aluminum box and build an instrument amplifier. The gain of the amplifier is 10000.

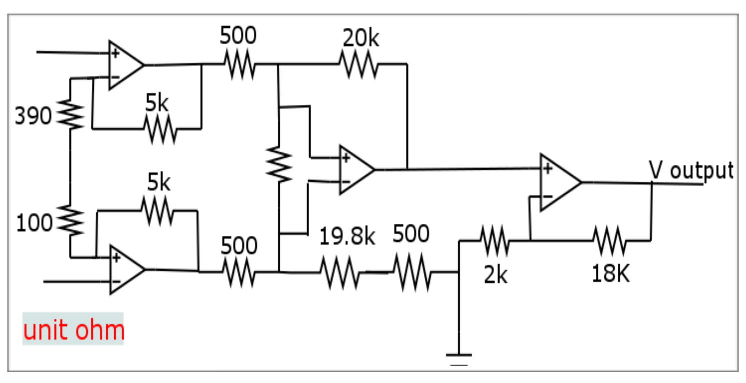

FIG.7 The circuit of homemade amplifier: the property of instrument amplifier has highly degree of symmetry, which makes a good signal-to-noise ratio.

Non-equilibrium steady state, NESS

- We make a bleeder circuit before the same parallel RC used in the first part.
- We injected a constant current about 1.53×10^{-13}A to establish NESS.
- The input power is about 47.21k_BT/s.

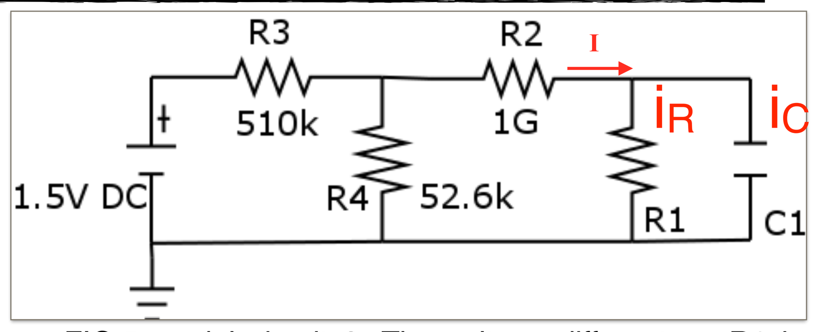

FIG.5 model circuit 2: The voltage different on R1 is measured.

By the Kirchhoff Circuit Laws, we can indirect get a energy conservation equation.

$$I = i_R + i_C$$

$$\Rightarrow \int_t^{t+\tau} V(t')I\ dt' = \int_t^{t+\tau} V(t')\, i_R(t')\ dt' + \frac{1}{2}C(V(t+\tau)^2 - V(t+\tau)^2)$$

$$\Rightarrow W_\tau = Q_\tau + \Delta_\tau V$$

The "Detailed Fluctuation Theorem" (DFT) is a modify form of FT when consider in the trajectory-dependent" entropy, ΔS_τ.

$$\Phi = ln(\frac{p(+\Delta S_{tot,\tau})}{p(-\Delta S_{tot,\tau})}) = \frac{<\Delta S_{tot}>}{k_B}, \forall\tau$$

where

$$\Delta S_{tot,\tau} \equiv \Delta S_{Q,\tau} + \Delta S_\tau$$

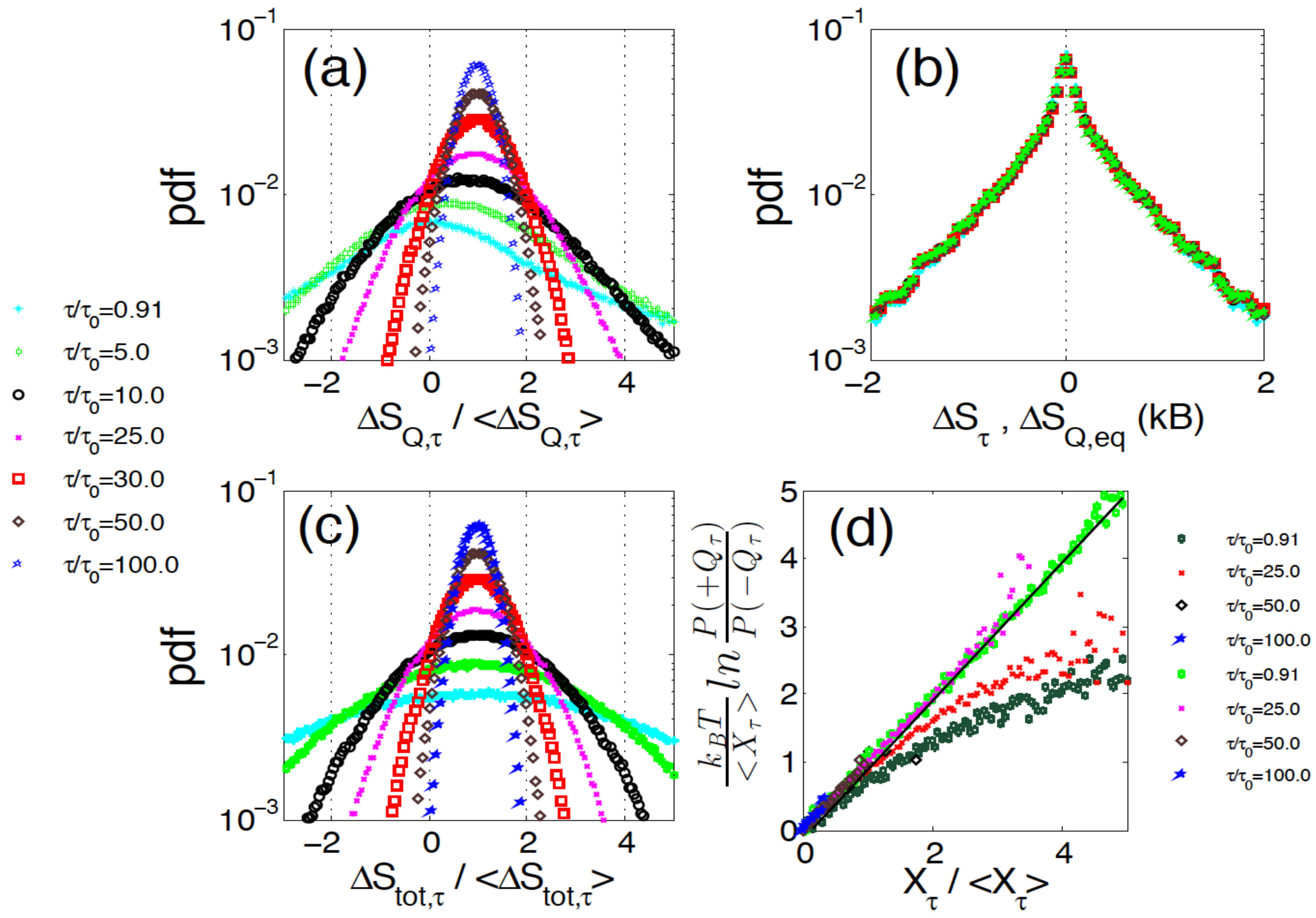

FIG.6. (a) PDFs of the normalized entropy of dissipative heat, $\Delta S_{Q,\tau}$ over different time lag, τ/τ_0. (b) PDFs of the "trajectory-dependent" entropy ΔS_τ. The distribution is independent to value τ/τ_0. (c) PDFs of the normalized total entropy, $\Delta S_{tot,\tau}$ over different time lag, τ/τ_0. (d) Symmetry functions Φ for $\Delta S_{Q,\tau}/<\Delta S_{Q,\tau}>$ at equilibrium state(the lower lines) and $\Delta S_{tot,\tau}/<\Delta S_{tot,\tau}>$ at NESS (the lines have slope near to 1) over the same τ/τ_0.

Discussion

1. In FIG.6 (a), we can see there are chances that the entropy production can be negative under small time span.
2. With the increase of τ, the probability density in negative area do decrease.
3. In FIG.6 (b), PDFs of "trajectory-dependent" entropy are independent to τ/τ_0.
4. In FIG.6 (c), PDFs of the total entropy is a Gaussian distribution.
5. In FIG.6 (d), we show how DFT modifies the FT. After we consider the influence of ΔS_τ , The slopes of the symmetry function of $\Delta S_{tot,\tau}$ are 1 in any time span.

Conclusion

- There is a fluctuation voltage in RC circuit, called Johnson-Nyquist noise.
- In microscopicy, the 2nd law of thermodynamics is not applicable, while we need the FT to modify it.

Reference

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