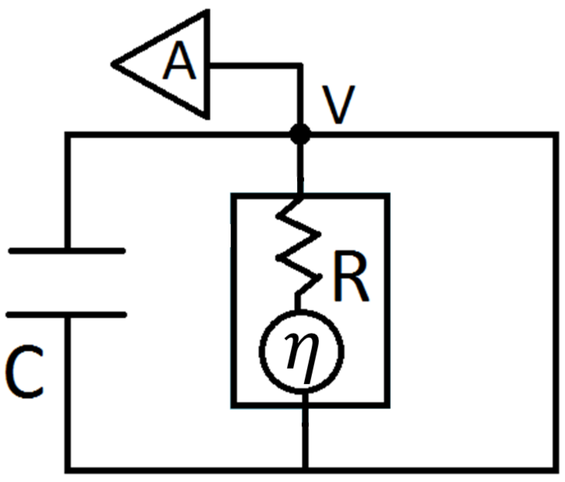
Equilibrium R-C circuit simulates Brownian motion

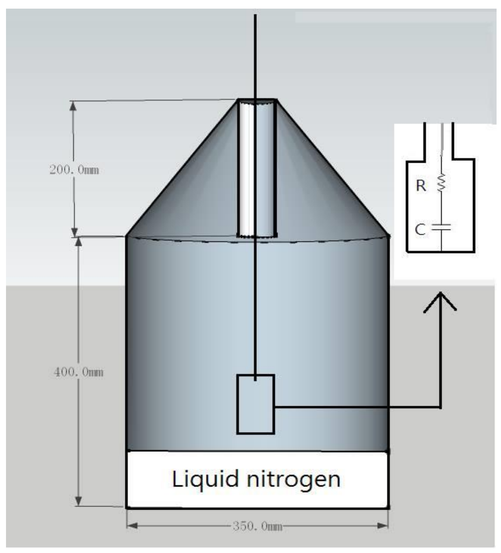
Kuang-Yu Chang(張光佑), Yi-Shang Tang(湯亦祥), Yi-Cheng Su(蘇翊誠)
Instructor: Yung-Fu Chen(陳永富), Kuan-Hsun Chiang(江冠勳)
National Central University

Abstract

We try to change the temperature of a RC circuit and observe the heat flux entropy and trajectory entropy. Then we discover the system is ruled by the same dynamics as Brownian motion and the noise of the circuit would be stronger if we raise the temperature.

Setup



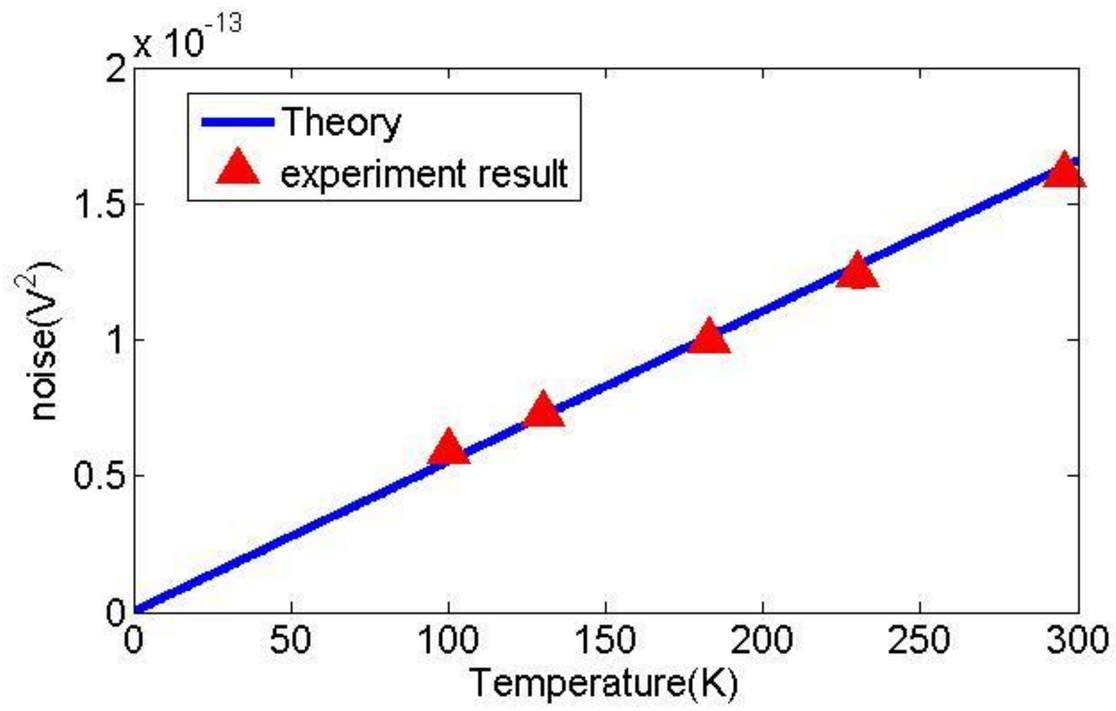


We make the system at different temperature with the Nitrogen liquid by adjusting the height of the box in the Dewar. $T = 300K$, 230K,180K, 130K, 100K. We check the temperature by the thermocouple and fix the height of the box with a shelf.

Johnson–Nyquist noise

$$|\eta|^2 = 4k_B RT$$

Johnson–Nyquist noise is electronic noise generated by the thermal agitation of the charge in the conductor, which happens regardless of any applied voltage. Without any power supply, we only see the Johnson–Nyquist noise whose intensity is proportional to the temperature(T) if we fix the resistance.

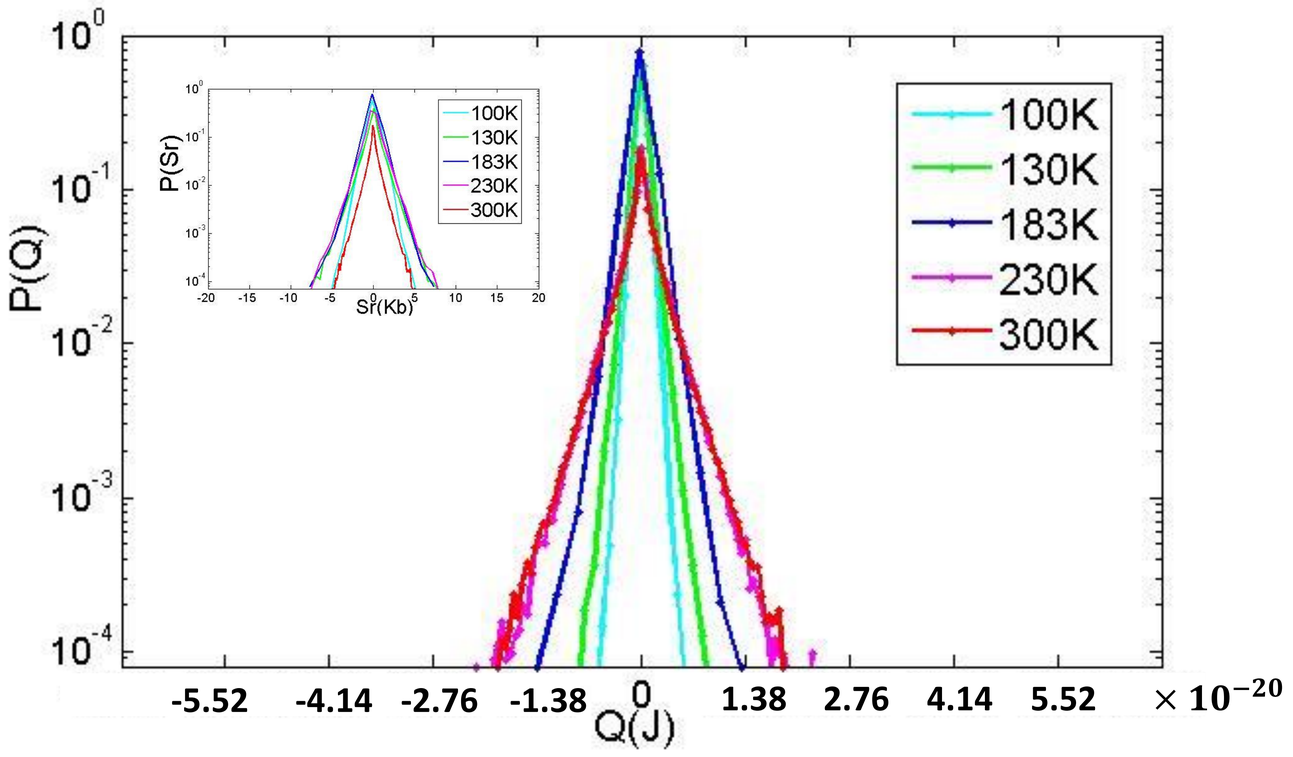


RC circuit and Brownian motion

$$\eta = R\dot{q} + q/C \quad , \quad \xi = b\dot{x} + kx$$

In our system, there are many characteristics similar with the Brownian motion system. Resistance is like the viscosity of the media. Charge is like the displacement of the particle. Current is like the velocity of the particle.

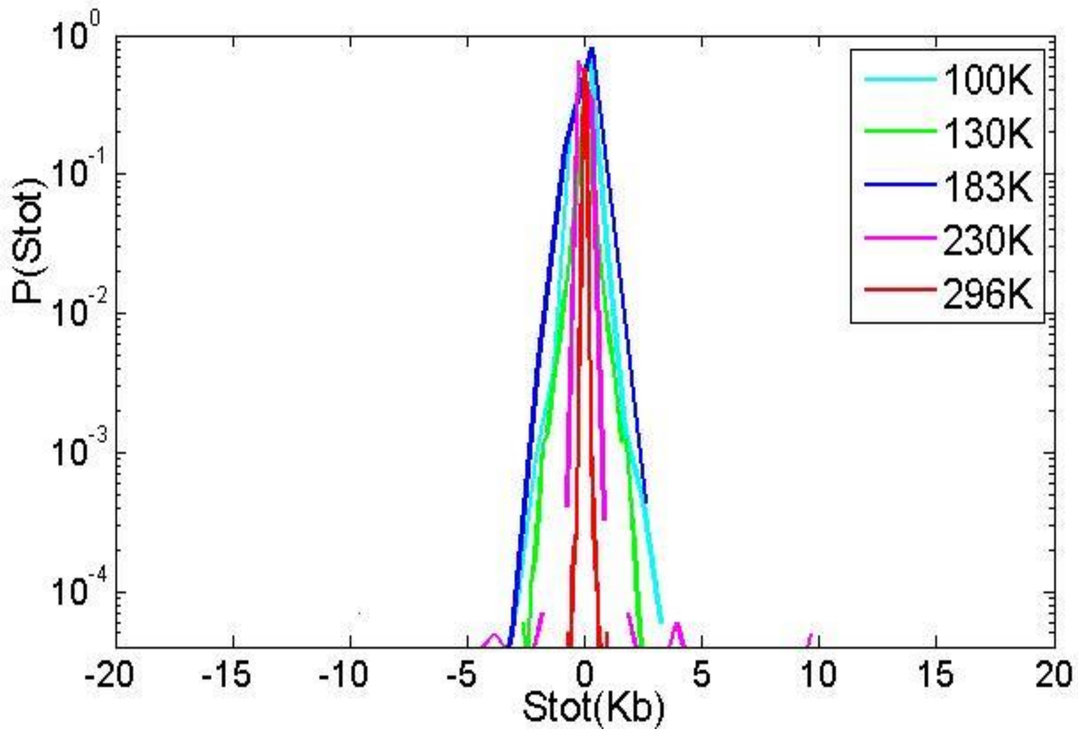
Heat flux



$$Q_\tau = W_\tau - \Delta U_\tau \ , \ \Delta S_{r,\tau} = Q_\tau / T$$

The heat flux between the resistor and the environment would be influence by the temperature. But we can see the temperature is independent of the heat flux entropy($\Delta S_{r,\tau}$) which caused by the heat flux. In different temperature, we can see their distribution are almost the same.

Total entropy



$$\Delta S_{s,\tau} = -k_B log \frac{P[V_1(t+\tau)]}{P[V_1(t)]} \qquad \left\langle \exp(-\frac{\Delta S_{tot}}{k_B}) \right\rangle = 1$$

When we consider the heat flux entropy and trajectory Entropy($\Delta S_{s,\tau}$) which comes from the state variable V, we can get the total entropy of this system. And the system in the steady state would correspond the above equation which is derived from the fluctuation theorem.

Conclusion

- The–heat noise is bigger if the system in an environment which temperature is higher.
- An RC circuit can simulate the Brownian motion in different temperature. Also, their total entropy always present an δ function in equilibrium state, $\Delta S_{total} = 0$.
- Johnson–Nyquist noise is surely proportional to the temperature when we fix the resistance.

Reference

1.N. Garnier *et al.*, *PRE* 71, 060101(R) (2005)
2.S. Ciliberto *et al.*, *PRL* 110, 180601 (2013)