Gravitational Acceleration Accurately Determined

Zheng-Hao Huang(黃政豪) , Yu-Chun Hsu(許鈺駿)

Instructor: Yung-Fu Chen(陳永富) , TA: Yu-Ting Lin(林侑鋌)

Experimental Physics Ⅱ, Department of Physics, National Central University

Motivation

- To measure the acceleration of gravity at a location as much precision as possible.
- To define a real pendulum is not so easily as a simple pendulum.
- To make a high precision measurement of g without too many complicated variables.

Goal

To get an exact value of g (five significant figures) by Kater's pendulum.

The value g should be...... $g \approx g_0(1 + 0.0052884\sin^2\emptyset - 0.0000059\sin^2 2\emptyset) - 0.000003086h$

Standard g	Calculated g	Height (m)	latitude (degree)
9.80665 m/s²	9.78860 m/s²	181	24.262

$$g \approx g_{45} - \frac{1}{2}(g_{90} - g_0)\cos 2\emptyset\,\frac{\pi}{180}$$

$g_0 \approx 9.78046\, m/s^2$

$g_{45} \approx 9.80665\, m/s^2$

$g_{90} \approx 9.83203\, m/s^2$

Principle

$\tau = -mgh_1 \times \sin\theta = I_1\frac{d^2}{dt^2}\theta$

$if\ \theta \to 0, then\ \sin\theta \to \theta$

$-mgh_1 \times \theta = I_1\frac{d^2}{dt^2}\theta$

$\theta(t) = \theta_0 \cos(\sqrt{\frac{mgh_1}{I_1}}t + \delta)$

θ_0: initial angle

δ: initial phase

$$T_1 = 2\pi\sqrt{\frac{I_1}{mgh_1}}$$

Parallel Axis Theorem

$I_{cm} = mR^2$ (R: Radius of Gyration)

$I_1 = I_{cm} + mh_1^2 = mR^2 + mh_1^2$

$$T_1 = 2\pi\sqrt{\frac{R^2 + h_1^2}{gh_1}} \quad T_2 = 2\pi\sqrt{\frac{R^2 + h_2^2}{gh_2}}$$

$$g = \frac{4\pi^2}{\frac{T_1^2 + T_2^2}{2(h_1 + h_2)} + \frac{T_1^2 - T_2^2}{2(h_1 - h_2)}}$$

$if\ T_1 = T_2 = T \quad if\ |T_1 - T_2| \to 0$

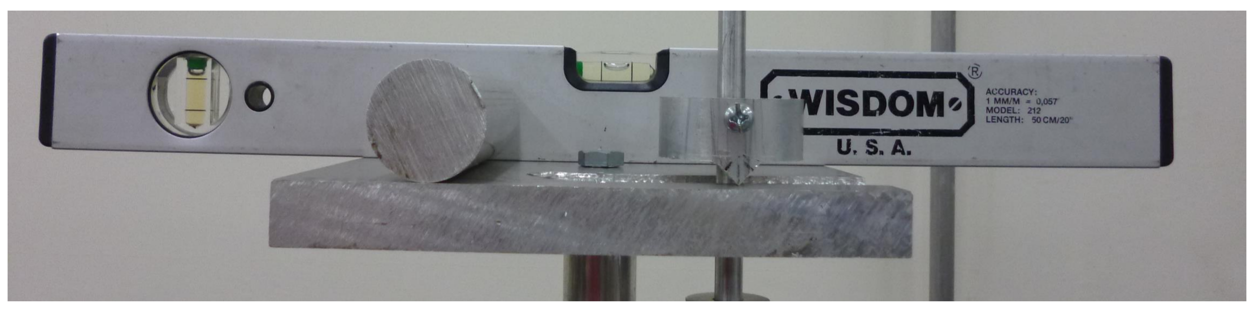
$$g = \frac{4\pi^2 L}{T^2} \qquad g = \frac{4\pi^2 L}{\frac{T_1^2 + T_2^2}{2}}$$

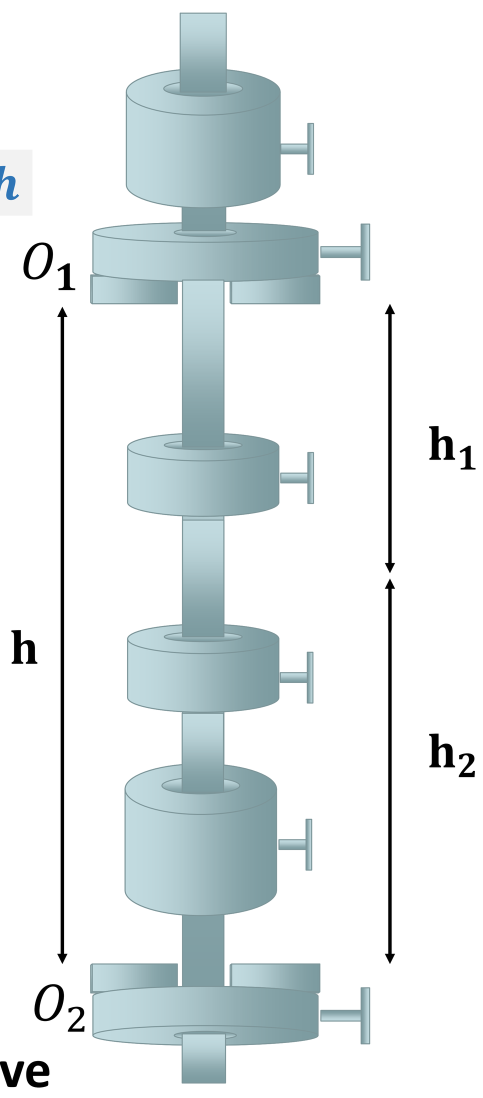
If one built an asymmetrical, double-ended pendulum, it could be adjusted so as to have exactly the same period T when swung from either pivot point.

When the condition is truly fulfilled, the effective length of the pendulum is just the distance L between the pivot points, which then can be measured by direct observation.

Setup

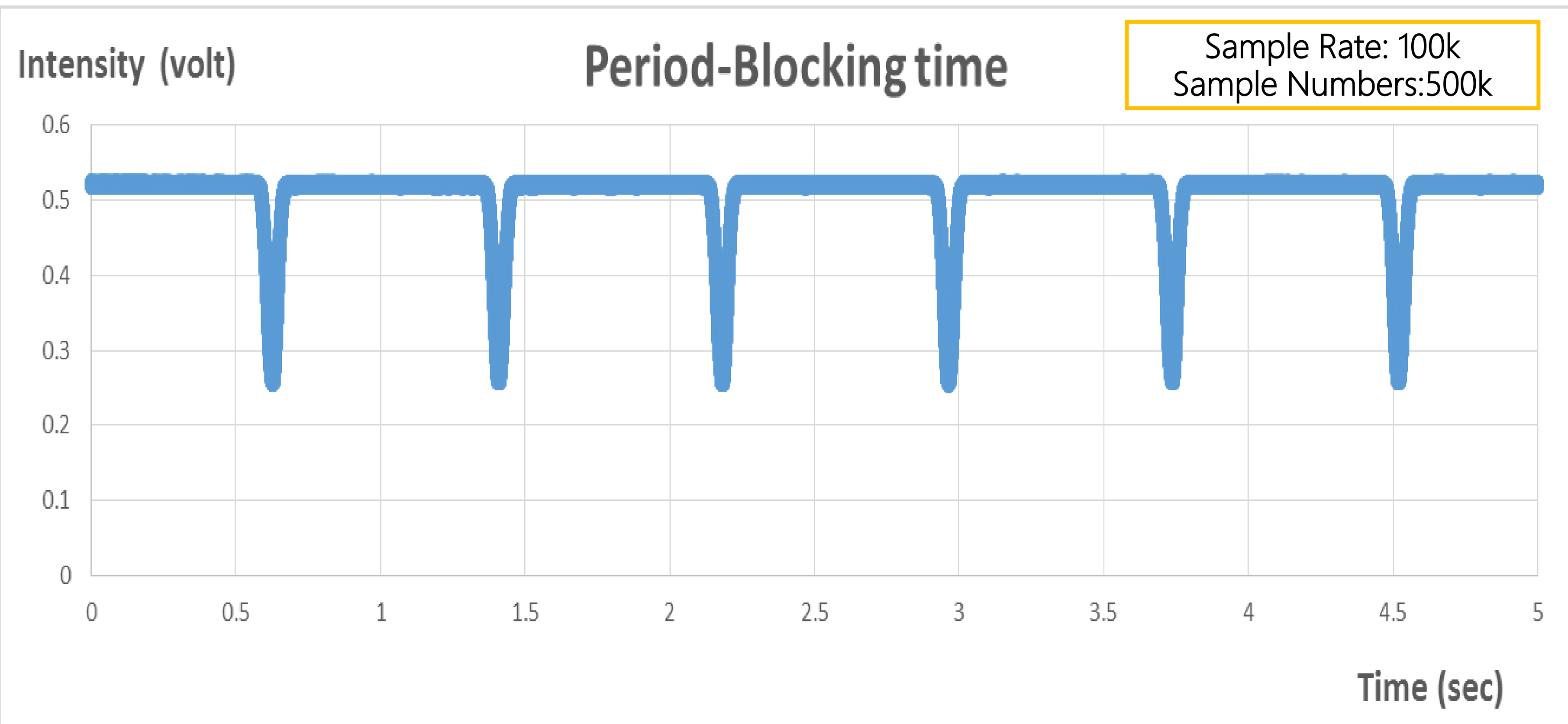
By Kater's Pendulum

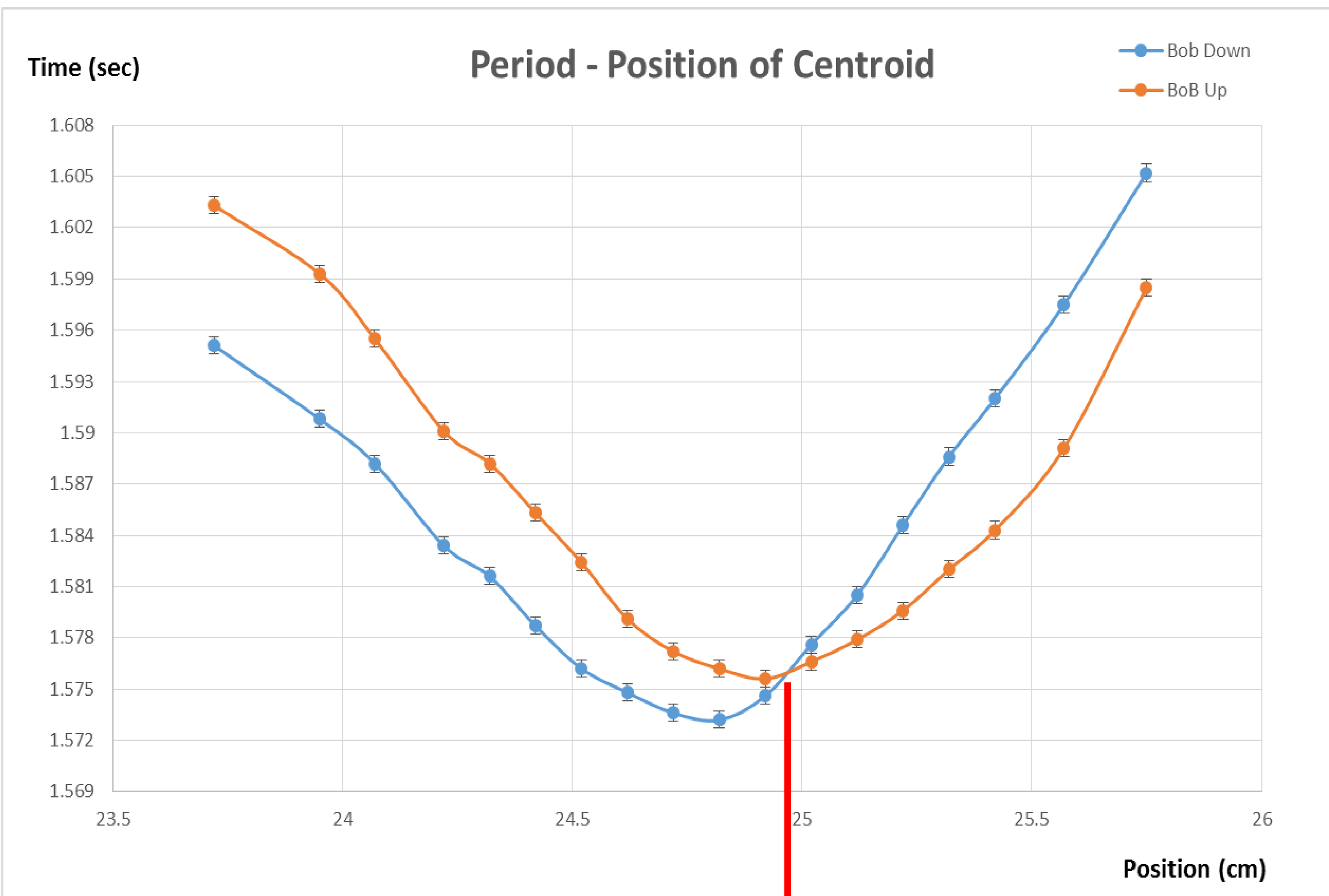




Result

Standard g (m/s²)	L (m)	T (sec)	g (m/s²)	error
9.7886	0.616070	1.5752	9.8021	0.138%
9.7886	0.616070	1.5750	9.8046	0.163%
9.7886	0.616070	1.5740	9.8170	0.290%
9.7886	0.616070	1.5754	9.7996	0.112%





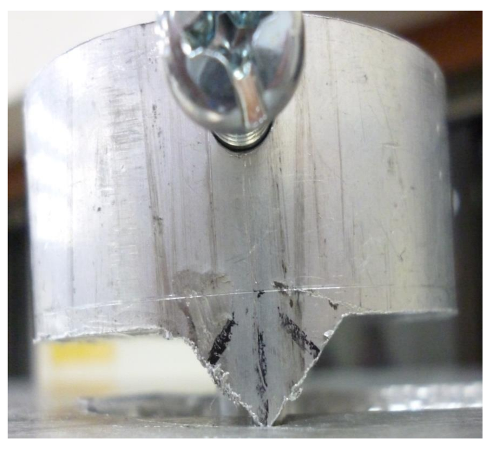
Calculated g
9.7886 m/s²

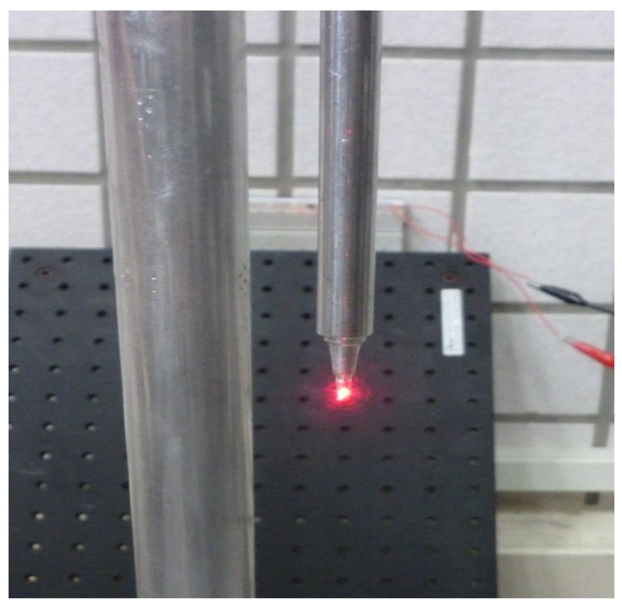
$L = 0.616070\ (m) \qquad g = 9.7884\ m/s^2$

$T = 1.5763\ \ (sec)\ error = -0.002\% \pm 0.001\%$

Length(m) → By Linear Scale + Touch Point Sensor

Time(sec) → By Laser: Laser ⇨ Photodiode ⇨ AD/DA Card ⇨ LabVIEW





Discussion

- Classical Mechanics
  Length & Time
- To reach the precise equal-period point
  Not practical
- Error caused by the factors below
  - Buoyancy of air
  - Damping of pendulum due to friction
  - Imperfect knife edges
  - Temperature variations during measurement
  - Elastic variations of pendulum length

Summary

- Setup a Kater's pendulum
- Eliminate the friction almost
- Try two methods to find the better mass combination of the bobs and the collars
- Determine Time and Length accurately
- Finally, we get the best value of g whose error is only -0.002%

Reference

- Kater, Henry (1818). "An account of experiments for determining the length of the pendulum vibrating seconds in the latitude of London". Phil. Trans. R. Soc. (London) 104 (33): 109. Retrieved 2008-11-25.
- D. Candela, K. M. Martini, R. V. Krotkov, and K. H. Langley. " Kater pendulum in the teaching lab " Physics Department, University of Massachusetts, Amherst, Massachusetts 01003.