Brownian motion

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Brownian motion was discovered in 1827 by Robert Brown. Today, the mathematic model was built and we could understand the physical meanings behind the formulas. Here we describe a simple experimental set-up to observe Brownian motion and a method of determining the Boltzmann constant, based on the result which come from solving Langevin equation.

As for our simulation in Matlab & R are straightforward, we solve the equation of motion, Langevin equation, by Euler method. Because of this simulation, we ensure easily that the experiment result conform the result of the simulations .

Introduction

Brownian motion is the first phenomenon studied exhaust in chaotic process. From 1827 to 1905, scientists were not able to formulate a theory to describe this zig-zag motion of dust in water. After the analysis of Brownian motion of Albert Einstein, the mathematic model was built and we could understand the physical meanings behind the formulas.

Here we introduce the Langevin formula to establish our simulation model.

In 1-D space, the Langevin equation shows

$$m\frac{d^2x}{dt^2} = f\frac{dx}{dt} - \eta(t) \qquad \langle \eta(t)\eta(t')\rangle = 2fk_bT\delta(t-t')$$

In this equation of motion, f is damping constant, 0.853 mpa·s. k_b is Boltzmann constant, 1.38×10^{-23} *J/K.*

$$\left\langle \frac{d}{dt}(x^2)\right\rangle = \frac{2k_BT}{\mu} \rightarrow$$ mean square displacement $<x^2>$ is proportional to t.

($\boldsymbol{\mu = 6\pi a\eta}$ for a Brownian particle of radius $\boldsymbol{a}$ in a solvent with viscosity $\boldsymbol{\eta}$)

In 2-D space

$$\langle R^2(t)\rangle = \langle {R_x}^2(t) + {R_y}^2(t)\rangle = \frac{4k_BT}{\mu}t$$

From the above formula, we can obtain $\boldsymbol{k_B}$.

Simulations Results

We use Euler method and Langevin equation to simulate this over damped motion before our experiments with matlab and R, we plot the MSD diagram(Fig2.) to help us understand this motion. MSD diagram can emerge diffusion of Brownian particles.

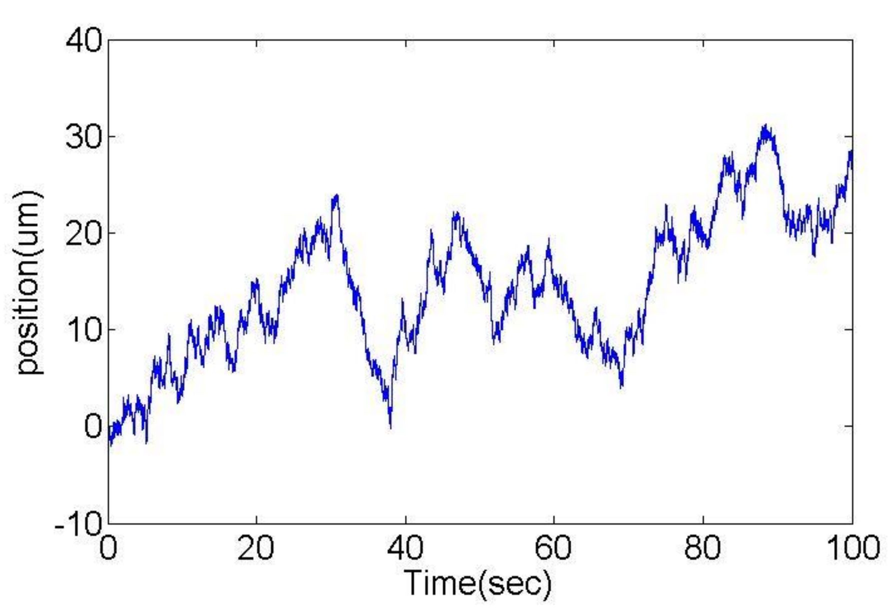

Fig1. 1-D Brownian motion. Parameter: $a = 3\ \mu m$, $f = 0.853\ mpa \times s$, *T=300 K,s*

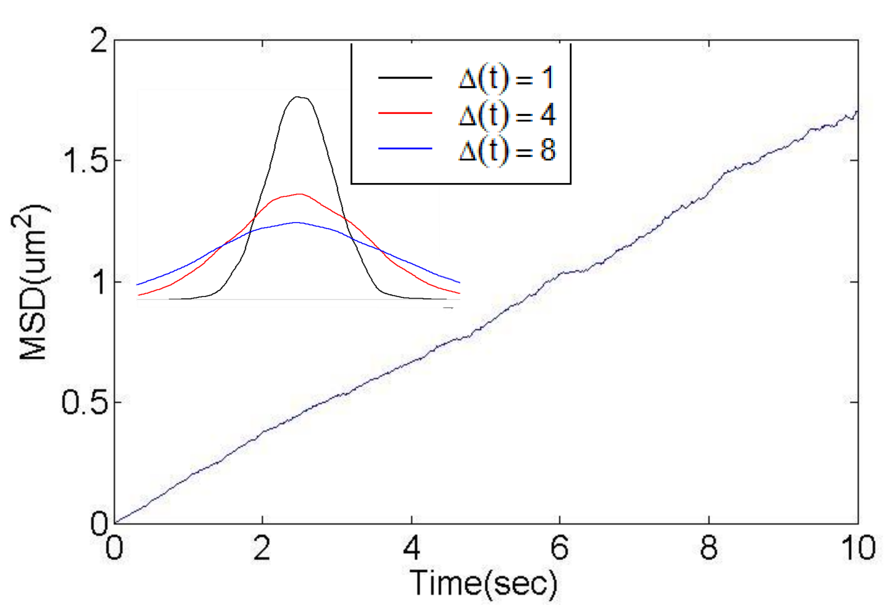

Fig2. MSD diagram. 1-D Brownian motion. Gaussian distribution with 1,2,4steps.

Method ＆ Experiment Set-up

Here we use Polystyrene as Brownian particles with diameter of 3 μm, and use a micro-titration needle to put a small portion of the particles into the solution where Glycerol and distilled water ratio is 2:11. We use cell phone flashlight instead bulbs to maintain a constant temperature(Fig4.). Next, we paste the tapes around the glass slide to keep the particles inside the central hole, and to prevent the flow, we wiped Vaseline around the coverslips(Fig3).

After the works done, we put the sample aside for about half hour, and the image would be captured by IC capture, and then being processed in ImageJ (Fig4.). Before fetching the data, we must perform the calibration of the image plane first. With sampling rate 76Hz, we took 650 images, recording time for around 8.5 seconds.

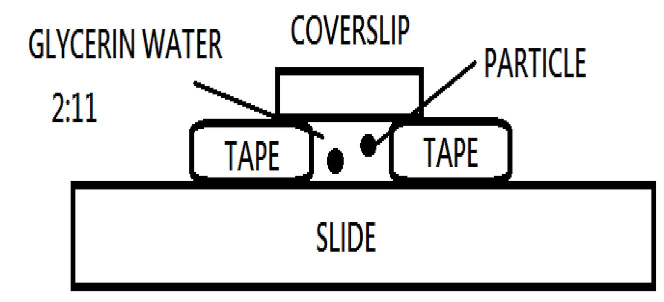

Fig3. Sample particles

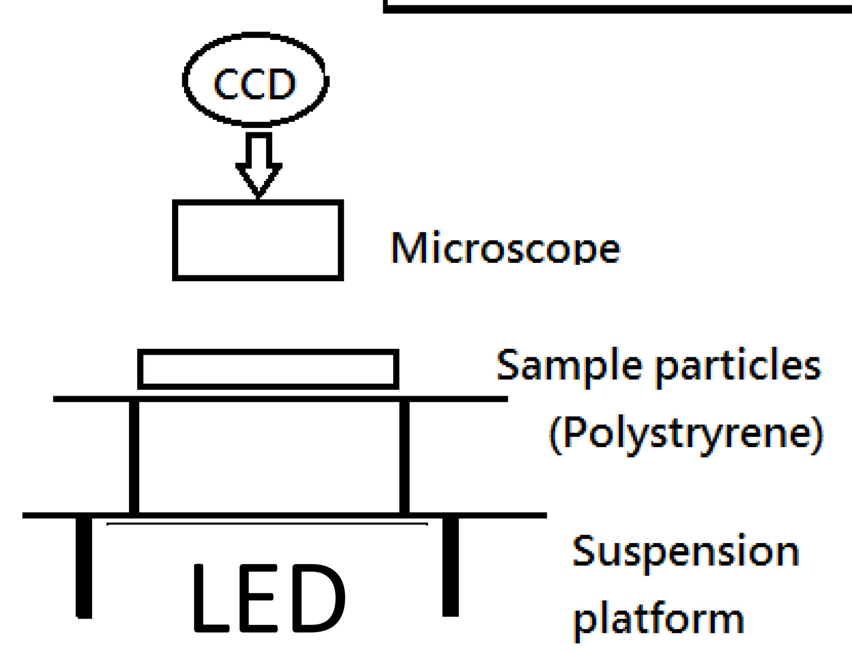

Fig4. Experiment set up

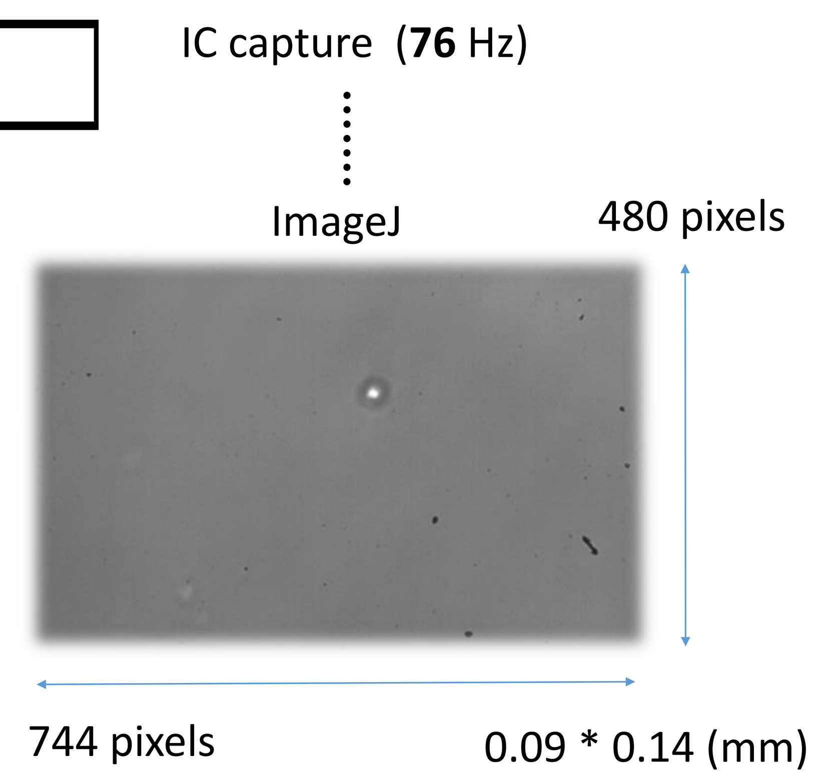

Fig5. Particle under the microscope

Estimation Results

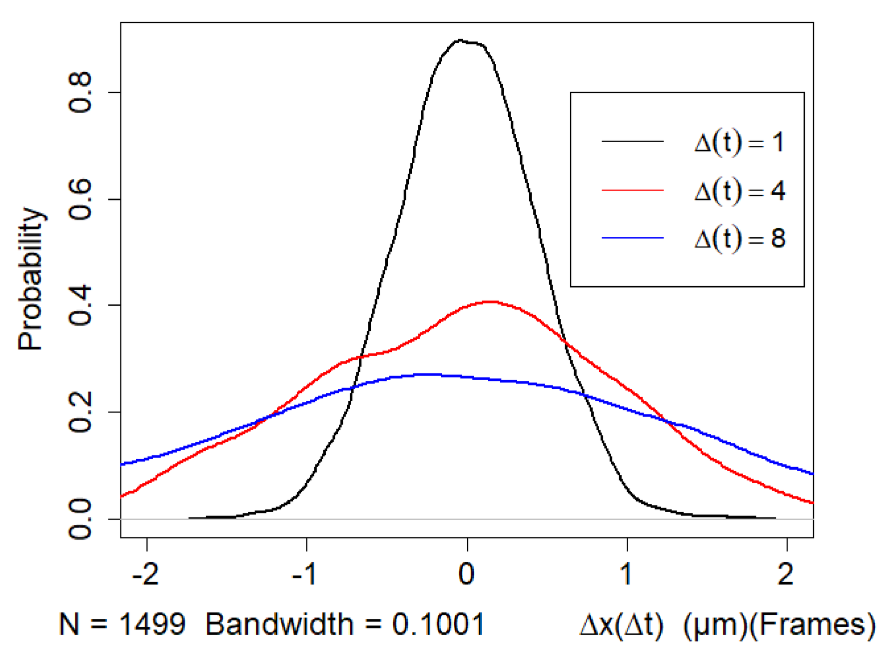

Fig6. Gaussian distribution with 1,2,4steps with 2-D Brownian motion, and FWHM in these steps are 1.089, 2.351, 3.435.

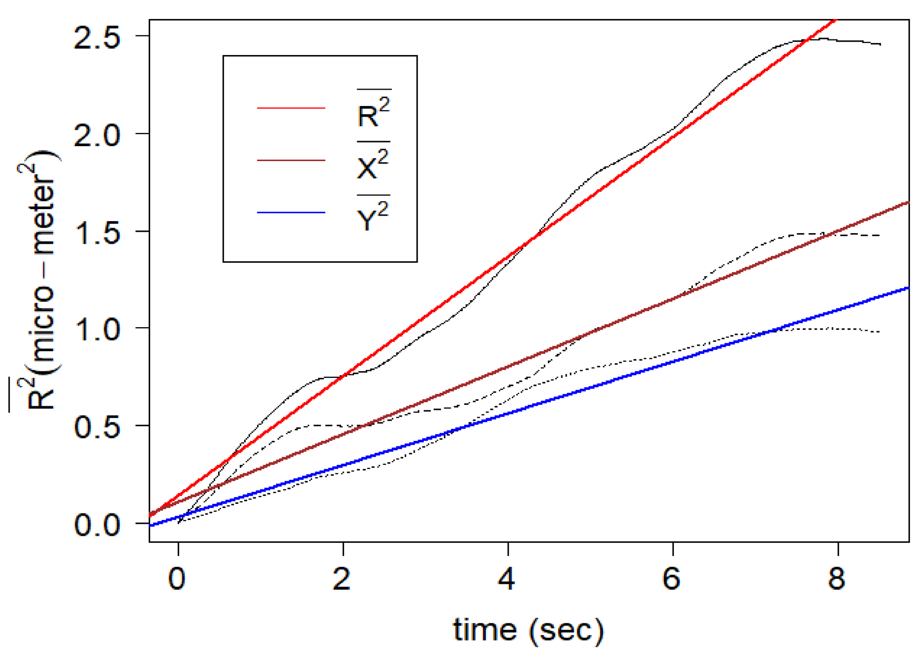

Fig7. MSD diagram. 2-D and x,y - direction Brownian motion.

We use the equation, $k_B = \frac{6\pi a\eta s}{4T}$ with the Parameter: $a = 3\ \mu m$, $\eta = 0.853\ mpa \times s$, *T = 297.4 K*, s come from MSD diagram (Fig7.), to calculate the Boltzmann constant.

In our experiment, we get $k_B = 1.244 \times 10^{-23}$,which is close to the theoretical number.

Conclusion

- The displacement of a particle is the Gaussian distribution, and the Gaussian distribution becomes flat as time increase.
- The mean square displacement of one particle is proportional to time, and the slope is relation to the Boltzmann constant.
- In our experiment, we ensure that the experiment result consistent with the result of our simulations .
- Next, we try to understand the relation between the temperature and the same particle on the MSD-plot in the experiment.

Reference

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