Dynamic of Elliptic Vortex ring

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Introduction

Vortex ring is a common phenomenon in nature, like a bobble ring created by a dolphin and smoke ring expelled from a smoke's mouth. Unlike circular vortex ring, elliptic vortex ring will have oscillatory deformation because it will have various induced velocity cause by different curvature.

In this study, we observed how the elliptic vortex ring deform. In addition, we fitted it by quadratic curve to compare circular vortex ring and the elliptic vortex ring in different aspect ratio.

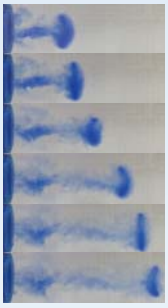


Fig. 1. Evolution of the elliptic vortex ring.

Experimental Setup

The nozzle can inject small amount of pigment ,let the vortex ring easier to be observed. The washer which is stuck to the balloon can be attracted by the electromagnet. Pulling back the electromagnet at fixed distance and turns off the power supply, the nozzle will generate a vortex ring.

When a vortex ring is generated, the water will come out quickly from the nozzle, which will bring out the pigment from the small gap in front of the nozzle.

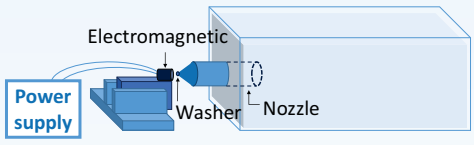


Fig. 2. Experimental setup in schematic diagram.

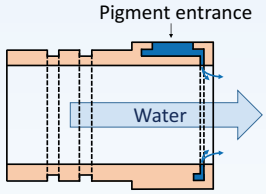


Fig. 3. The schematic diagram of nozzle in sectional view.

Theory

When injecting a group of fast moving fluid into a stationary fluid, the interface between two fluid in different velocity will have viscous friction. It will slow down the outer layer of fast moving fluid and try to rotate the surrounding fluid.

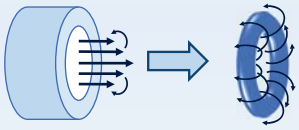


Fig. 4. Evolution of the vortex ring in schematic diagram.

The velocity d**v** caused by the curved vortex filament d**l**, which gives the Biot-Savart Law by analogy:

$$d\mathbf{v} = \frac{\Gamma}{4\pi}\frac{d\mathbf{l}\times\mathbf{r}}{r^3}$$

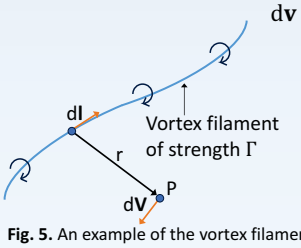


Fig. 5. An example of the vortex filament.

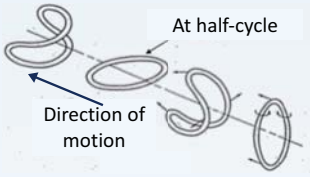


Fig. 6. The trajectory of elliptic vortex ring in schematic diagram [1].

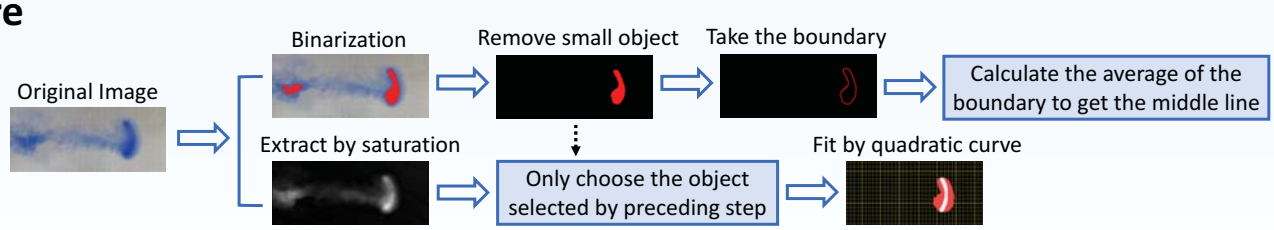
Consider a general vortex filament, which has a constant strength Γ. And we can get a finally induced velocity

$$V \propto \frac{\Gamma k}{2\pi}$$

k: value of the curvature

Therefore, the elliptic vortex ring has a variable curvature in different part, so that the vortex ring will deform during the motion [2].

Procedure



Results and Discussion

Trajectories of the vortex rings

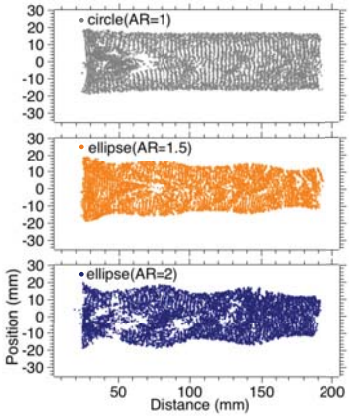


Fig. 7. Trajectories of the vortex rings.

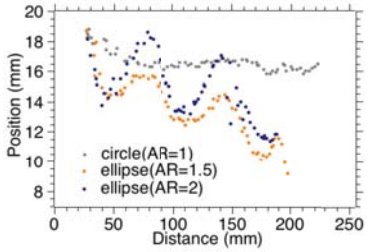


Fig. 8. Trajectories of the top boundary.

- The amplitude of elliptic vortex ring will increase as the aspect ratio increasing.
- The pigment of elliptic vortex ring will diffuse quickly because of oscillation.

The zero term and the position of center of mass

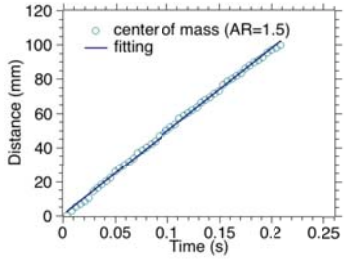


Fig. 9. Position of the center of mass.

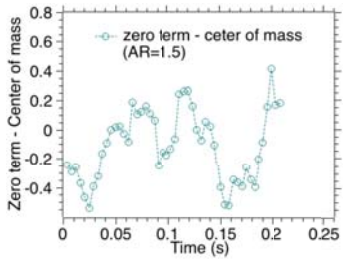


Fig. 10. Zero term - position of the center of mass.

- The zero term is influenced by the oscillation.
- There is an oscillation while zero term minus center of mass.

The second term of the quadratic curve

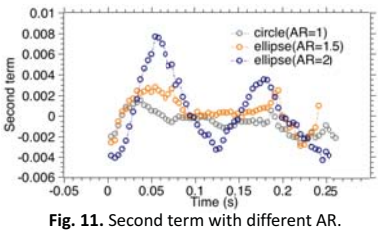


Fig. 11. Second term with different AR.

- The maximum value of the second term of elliptic vortex ring with AR=2 will be the largest.
- The second terms is in periodicity.

The second term in different view

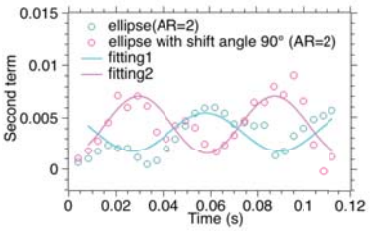


Fig. 12. Second term in different view.

- Compare the second term in side view and the one in top view, their major axis and minor axis are exchanged.

Conclusions

- The amplitude of vortex ring and its bending extent becomes larger as aspect ratio increase.
- The elliptic vortex ring will exchange its major axis and minor axis during the motion due to the change of the curvature as time goes on.

References

[1]Adhikari, D. Some experimental studies on vortex ring formation and interaction. Master thesis. Singapore: Department of mechanical engineering, national university of Singapore (2009).

[2]Bai, R. Study on the vortex ring and its phenomena. Term paper. Cambridge: Harvard university.